Vote or Fight?*

David K. Levine[†], Cesar Martinelli[‡], Nicole Stoelinga[§]

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Abstract

Why do some nations allocate power through voting while others do it through fighting? When political power is indivisible, voting is a substitute for fighting—provided the losing side accepts the outcome. We study a theoretical model of this substitution, and use a country-level panel dataset to assess empirically whether the economic factors influencing fighting also shape voting. They do. We contribute several theoretical and empirical innovations. First, we apply a recently developed method for analyzing conflict resolution functions to derive robust theoretical results. Second, we introduce a new explanatory variable—productive efficiency as measured by income relative to the global frontier—and we explain the theoretical and empirical relevance of this variable. Finally, we show that the absolute level of income does not matter, while oil wealth and ethnic division do - but that they are less important than productive efficiency in explaining fighting and voting. A key implication of our analysis is that reducing global inequality is crucial for decreasing conflict and fostering democracy.

Keywords: game theory, democracy, civil war, conflict, voting

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[†]RHUL Economics and Emeritus WUSTL; david@dklevine.com

[‡]Economic Department, George Mason University; cmarti33@gmu.edu

[§]Max Planck Institute for Research on Collective Goods; stoelinga@coll.mpg.de

1. Introduction

Why do some nations adopt peaceful and democratic institutions and others do not? To address this question, we develop a theoretical model where voting and fighting are treated as substitutes. The model implies that the same economic fundamentals influencing the prevalence of fighting should also shape the prevalence of voting. Using data on civil conflict, voting, and income, spanning 1815 to the present, we find strong evidence supporting this idea. Specifically, our findings reveal that productive efficiency measured by per capita income relative to the global frontier—not the absolute level of per capita income—plays a critical role in determining both fighting and voting. The main conclusion of this paper is that fostering peaceful and democratic institutions requires promoting economic efficiency to reduce global inequality, rather than attempting to reform political institutions directly.

The first innovative element of this paper is the use of a single model with common parameters to explain both the incidence of fighting and the incidence of voting. The second is the use of productive inefficiency as an explanation for both. As indicated, we find that productive inefficiency is an important explanation of fighting and that the parameters estimated from the fighting model provide a good explanation of voting.

There is a great deal of prior work explaining either fighting or voting, and a great deal of prior work using per capita income as an explanation. There is, however, no work that attempts to explain both fighting and voting with a single model, nor has productive inefficiency been used as an explanation for either. Some have concluded that per capita income is a poor explanation of either fighting or voting, and that ethnic divisions are more important. Our work supports the conclusion that per capita income is a poor explanation and that ethnic divisions, as well as oil and natural gas wealth, are important, but we also show that productive inefficiency is more important.

Our framework focuses on the contest between two parties vying for political power. A key assumption is that political power is difficult to divide—one party must ultimately win. Strength, however, can be measured in two distinct ways: through voting or fighting. Crucially, elections are meaningful only if the losing party accepts the results and refrains from fighting: Thus, our point of departure is that voting and fighting are substitutes. We analyze this substitution to assess the likelihood that power is allocated through fighting—defined as attempts to seize power through force—versus voting, where the loser respects the outcome.

Our model shows that incumbents are more willing to fight than opposition parties. If both parties are willing to fight, power is allocated through fighting. If only the incumbent is willing to fight, the opposition concedes, resulting in a dominant party system. If neither party is willing to fight, power is allocated through voting. Fighting is a costlier method of allocating power because, in addition to the effort expended, it incurs battle damages. These battle damages depend on productive inefficiency—a key explanatory variable in our model. Economically inefficient nations experience lower battle damages, making fighting more likely and voting less likely.

Our model offers two key innovations. First, we build on recent theoretical developments, notably Ewerhart (2017), showing when the mixed strategy equilibria of contests are unique, and the equilibrium characteristics—probability of winning and expected effort— are independent of the conflict success function's specifics. The study of conflict has been previously handicapped by the fact that the marginal increase in the probability of winning tends to be larger when the two parties are more evenly matched and the resulting equilibrium is in mixed strategies; these equilibria typically are difficult to compute and highly dependent on the details of the conflict success function (see, for example, Ashworth and Bueno De Mesquita (2009)). Second, we provide a framework linking income to the probabilities of both fighting and democracy, enabling empirical validation against historical data.

Our empirical analysis focuses on the role of income in determining whether nations resort to voting or fighting. Income is widely recognized as a robust predictor of civil war, and our data confirm its significance for both voting and fighting. However, our data uncovers a key distinction between temporal and cross-sectional effects. Despite significant global income growth, the probability of fighting has not decreased substantially over time. If anything, it has increased. This is shown in Figure 1. The dots in the figure are the probability that outcomes are determined by fighting, and the curve is normalized world per capita GDP. In section 11 we provide more systematic evidence that rising per capita GDP has little impact on the incidence of fighting.

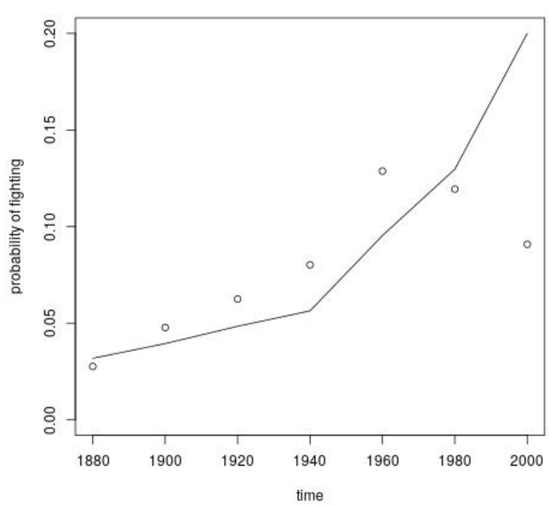

Figure 1: Fighting versus time
Dots indicate the probability of fighting. The curve represents world per capita GDP normalized to be equal to 0.20 in 2010. Data sources are provided in section 6.

In contrast, countries closer to the global productivity frontier experience a markedly lower probability of fighting. To capture the cross-section we examine the relationship between the probability of fighting and a measure of productive inefficiency: the ratio of per capita GDP on the frontier to that in the country. This is shown below in Figure 2 where the dots show the probability of fighting.

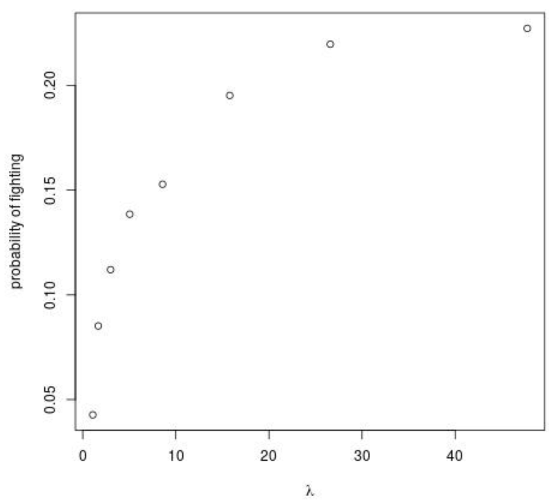

Figure 2: Probability of Fighting
λ is the lagged productive inefficiency as described in section 6. Dots are probability of fighting aggregated into cells.

Our model attributes these patterns to a benefit cost analysis. Both voting and fighting entail costs—mobilizing voters or soldiers—but fighting incurs additional battle damage: by our estimate \$100 spent on soldiers creates around \$2000 of damage. To study this battle damage, we introduce a simple benchmark model of how battle damage varies with per capita GDP. Just as the gravity model has proven a useful benchmark for studying international trade, we think our simple benchmark model of productive efficiency can prove useful for studying political economy.

Specifically, we propose to model the standing of a country relative to the frontier technology. The idea that catching up to the frontier is different than moving the frontier is widely used in studying technological change (see, for example, Kortum (1997)). A country that it not at the frontier takes more time to produce the same output than the frontier country. The effect of battle damage is to increase that length of time. In other words: a poor country looks like a rich country with battle damage. Think of the frontier country as a finely tuned automobile engine. Other automobiles have sand and debris in the engine that lead them to operate less efficiently. Throwing more sand in these engines has much less impact than on the finely tuned frontier engine. In other words: as inefficiency increases, the additional harm done by battle damage is diminished. This means that the benefit cost analysis for inefficient countries is more favorable to fighting.

The benefit cost analysis is also different for an incumbent than for the opposition: the incumbent has a substantial advantage and is consequently more willing to fight. This means that there are three possibilities: if the opposition is willing to fight then neither party is willing to abide by the result of the election and there will be a fight. If the incumbent is not willing to fight, neither party is willing to fight and there will be voting. Finally, if the incumbent is willing to fight and the opposition not, then the incumbent will remain in power without a meaningful vote: there will be a dominant party.

With these ingredients we assume that countries differ only in their productive inefficiency. We then estimate the structural parameters of the model by assessing the impact of productive inefficiency on fighting. Using the parameters estimated we ask whether those parameters also explain the incidence of voting. Figure 3 shows that, indeed, using the model and only data on fighting we can well predict the probability of voting.

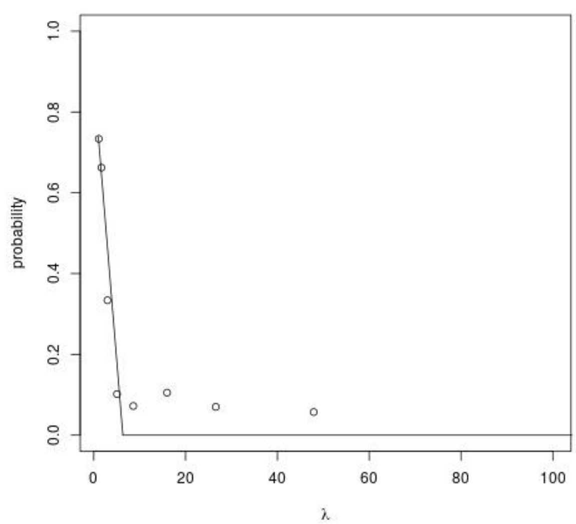

Figure 3: Probability of Voting
λ is the lagged productive inefficiency as described in section 6. Dots represent voting data aggregated in bins shown in Table 3. The solid line represents voting probability predicted from data on fighting.

Having a theoretical model is important for assessing data. As an example, Acemoglu et al. (2008) argue that empirically income does not lead to democracy. Generating data from our model we can use the Acemoglu et al. (2008) procedure on our artificial data. Although we know that causality in this data runs from income to democracy, we find exactly the same empirical results that Acemoglu et al. (2008) argue proves the opposite.

We also assess the impact of variables such as oil income and ethnic division due to language differences on both fighting and voting. These are consistent with the model parameters, and as expected both increase fighting and reduce voting substantially. We find however, that these effects are smaller than the impact of productive inefficiency.

Our findings underscore the importance of reducing global inequality to foster democracy and reduce conflict. Helping nations move closer to the productivity frontier reduces the relative attractiveness of fighting, paving the way for democratic governance. Promoting economic efficiency—e.g., by reducing trade barriers for inefficient nations—can enhance both economic and political outcomes. This is particularly the case, since the other important explanatory factors - oil income and ethnic divisions due to language differences - are not easy to change.

The remainder of this paper is organized as follows. The relation with previous literature is discussed in section 2. The model is described in section 3. The main result and its proof are provided in sections 4 and 5. Our empirical approach is described in section 6. Our structural estimation of the parameters of the contest success function and of the effect of productive inefficiency on fighting are described in sections 7 and 8. In section 9, we compare the model predictions on the likelihood that countries award power through voting with the data. In section 10, we show that our model is consistent with empirical work claiming that the causation goes from democracy to economic fundamentals. In section 11, we assess the impact of variables such as absolute per capita income, oil income and ethnic divisions on fighting and voting. Section 12 gathers concluding remarks.

2. Relation to Previous Literature

We want to emphasize that our empirical analysis is theory driven and why this makes a difference. First, our definition of both fighting and voting differ from standard definitions of civil war and democracy. In particular fighting means attempting to overthrow the government by violence. This excludes regional civil wars but includes coups and coup attempts that do not involve widespread violence. Voting means an election in which the incumbent may lose and will allow the opposition to take power if it does. It does not depend on the extent of the franchise as in Acemoglu and Robinson (2001). In particular while, for example, the enfranchisement of women in the US and UK improved democracy immeasurably, it did not change the fact that power was already being passed peaceably from one party to another through elections.

Second, theory tell us about measurement. Probably most important is it tells us that it is per capita GDP relative to the frontier that matters not absolute per capita GDP. It tells us how long periods should be: the length of period should be the time a government is typically in power before facing reelection. (As a practical matter this is five years.) Finally, it tells us that during five year periods we should measure five year GDP as an average, not as a single year out of the five. The reason is that GDP matters because of the expectations it creates for the parties about the costs and benefits of conflict. Last year's unusually large or small GDP is scarcely reason to change these expectations.

Our work here contributes both to the literature on civil war and that on democracy. We discuss the literature on the relationship between income and both democracy and civil war below in section 10. Fearon (1995) provides a classical analysis of the sources of civil war and it is useful to place our model into his framework.

- Bounded rationality: this is present in our model in the form of random optimism and pessimism about battle damage. In our model, fighting is driven by optimism and voting by pessimism.

- Agency problems: this is not part of our model.

- Asymmetric information: this is not part of our model. For an empirical model that does incorporate asymmetric information, see Dal Bo and Powell (2009). This is a signaling game in which the government has private information about the size of the spoils. Like our model, this leads to a theory in which the probability of fighting is higher when income is low. In Laurent-Lucchetti, Rohner and Thoenig (2024) ethnic groups negotiate over economic surplus, and asymmetric information regarding the strength of the opposition can lead to bargaining failure and civil conflict. Much of the work on asymmetric information, such as Corchon and Yildizparlak (2013), is purely theoretical. Particularly relevant is Fearon (2011) where, as here, voting serves as a substitute for rebellion. The focus in that paper is on the use of elections as a coordination device for rebellion. Our assumption that if the opposition does not concede and an election is not held they are committed to fight can be thought of as a reduced form of the Fearon model.

- Commitment problems: this is implicitly present in our model—it is essential that the parties are not be able to commit to respecting the results of an election. In Przeworski, Rivero and Xi (2015) the commitment is explicit with the incumbent tailoring the probability of winning to avoid conflict.

- Indivisibilities: this is our main assumption, that power cannot be shared. See, for example, Hirshleifer, Boldrin and Levine (2009) on why this is so.

Some of this previous work has incorporated as do we the idea that fighting and voting are substitutes. This is especially the case in the literature on democracy: Laurent-Lucchetti, Rohner and Thoenig (2024) and Przeworski, Rivero and Xi

(2015) are models in which voting is an alternative to fighting. Neither paper, however, studies the implications of the model for fighting.

There is a literature using temperature as an instrumental variable to explain fighting. Miguel, Satyanath and Sergenti (2004) use this method to argue that economic growth depresses fighting. However, Dell, Jones and Olsen (2012) show that there are difficulties replicating this result and argue that the incidence of fighting is not well explained by temperature, but that the incidence of civil unrest is. There are several issues with this literature from our point of view, including the absence of any theoretical rationale for these estimations. First, our theoretical model argues that it is distance to the frontier, a level, that should matter, not growth. Second, the notion of fighting in these models includes all fighting, intra-state as well as inter-state, and there is no reason to think that the two are triggered by the same underlying conditions. Finally, the civil unrest variable includes only civil unrest that results in a change of government—successful coups but not unsuccessful coups, while theory indicates that the underlying sources of both is the same, while the outcome is random, but related in a different way to the underlying economic factors. Notice that the issue dealt with by instrumental variables—the fact that growth is endogenous—we deal with by using productive inefficiency prior to the outbreak of fighting. Our approach is discussed in section 6.1.3.

Some have concluded from Dell, Jones and Olsen (2012) that economic growth is not an important explanatory factor in conflict and have turned toalternatives beyond income. Esteban, Mayoral and Ray (2012) bring a theoretical model of polarization due to ethnicity to study data on ethnic conflict. We use their data and show that indeed ethnic division is important in explaining fighting. There have been several studies concerning the relationship between trade and civil war: Besley and Persson (2008) focus on export and import prices, while Dal Bo and Dal Bo (2011) study a general equilibrium model. Those interested in this interesting and diverse literature will find the excellent survey article of Blattman and Miguel (2010) a helpful guide. Our focus on alternative economic factors looks at the role of oil and natural gas. For both ethnic division and oil and gas we show that while they are important explanatory variables, productive inefficiency is more important.

3. The Model

There are repeated games between two parties: in each period one party is the incumbent party i and the other is opposition party o. As a result of the game their roles may be the same or reversed in the next period. Specifically, both parties are myopic and in each period there is a game determining which party will take power. The winning party is the incumbent in the next period and wins a prize in the current period that both parties value equally.

First a random shock ϵ, the *degree of pessimism*, i.i.d. over time, is drawn and is common knowledge. Then the opposition moves and may either *challenge* the incumbent or *concede*. If the opposition concedes the game ends and the incumbent party remains in power. (There may or may not be a non-credible "show" election: this does not matter to the game.) If the opposition challenges then the incumbent moves and may either hold a credible *election* or trigger a fighting. In other words, the challenge by the opposition is a credible commitment to fight if a credible election is not held. If a credible election is held it results in a winner and loser as described below. The loser then has a final opportunity to fight.

If there is an election the winner is determined stochastically depending on the *electoral effort* $e_k \geq 0$ provided by each party $k \in \{i, o\}$. If there is fighting any election results are discarded and the winner is determined stochastically depending on the *fighting effort* of the two parties $f_k \geq 0$.

For empirical purposes we will compare different countries at different times. We take the fundamental economic difference between countries to be per capita GDP, which we denote by γ. This serves as a scaling factor for the benefits and costs of conflict. Specifically, both groups value the prize equally as $\gamma V > 0$ per capita where $0 \leq V \leq 1$. Effort is costly to both groups. The expected marginal cost of effort for group k is γB_k and γC_k for the election and fighting respectively. Note the assumption that in a higher income country the value of the prize is proportionately higher, but so is the opportunity cost of providing effort for a conflict. The realized expected direct cost to group k is $\gamma B_k e_k + \gamma C_k f_k$. In addition fighting creates costly battle damage: group k suffers an additional expected cost of $d(f_{-k})$ depending on the effort of the other group. We will provide more details about the damage function $d(f_{-k})$ below. We assume that the incumbent has an advantage in the contests in the sense that $B_i < B_o$ and $C_i < C_o$.

The outcome of each contest is determined by effort according to contest success functions. Specifically, we assume that the probability of k winning the contest does not depend on the scale of the conflict, only on the relative effort of the two parties. Hence, for $e_{-k}, f_{-k} > 0$ and $e_k \leq e_{-k}$ and $f_k \leq f_{-k}$, the probability that k wins is given by $P(e_k/e_{-k})$ for voting and $Q(f_k/f_{-k})$ for fighting respectively. As these functions depend only on relative effort they should satisfy $P(1) = Q(1) = 1/2$. We assume that greater effort leads to greater success, so that these functions are weakly increasing and following Hirshleifer's (1989) we assume that a small amount of additional effort is more likely to make a difference in a close contest than a one-sided one, so that these functions are weakly convex. Note that these assumption imply continuity on $[0, 1)$. More strongly, we requires that where the functions are strictly increasing they are strictly convex. It will be convenient to abbreviate $P_0 = P(0), Q_0 = Q(0)$. If neither party provides effort each has an equal chance of winning.

Three examples of contest success functions $H(x)$ satisfying our assumptions[1] are translations of Tullock's (1980) function $H_0 + (1-2H_0)\left(1/(1+x^{-\alpha})\right)$ with $\alpha > 2$, translations of the serial contest success function $H_0 + (1 - 2H_0)(1/2)x^{\alpha}$ with $\alpha > 1$ studied by Alcalde and Dahm (2007), and transla-

[1]Bevia and Corchon's (2015) contest success function depends on the ratio but is concave so does not satisfy our assumptions.

tions of the all-pay auction in which the probability of winning for $x < 1$ is H_0, where in each case $0 \leq H_0 \leq 1/2$.

We assume that $P_0 > 0$ meaning that if there is an election then there is some chance of success regardless of effort. As a practical matter we believe that there is: in particular, we believe that there is a difference between not contesting an election and providing no effort. If there are two candidates on the ballot, regardless of effort, unusual circumstances may intervene. For example, in January 1986 the Democratic presidential hopeful Gary Hart was polling nearly 46%. His closest rival, Mario Cuomo pulled out of the race and he retained a commanding lead over his Democratic rivals until in May 1987 when photographs of himself with scantily clad women who were not his wife appeared in a number of newspapers, and he withdrew from the race. As a result Michael Dukakis became by default the Democratic candidate, and the Republican nominee, George H.W. Bush won in a landslide. Although in 1983 Edwin Edwards said "The only way I can lose this election is if I'm caught in bed with either a dead girl or a live boy" stranger things have happened.

Our notion of equilibrium is subgame perfection.

3.1. Productivity and The Damage Function

Different countries at different times have different levels of productivity. A key component of the model is how opponent effort creates damage in the form of lost output. We adopt a simple model of productivity differences. We imagine that at any moment of time there is an overall technology parameter representing the frontier economy: denote this by $g > 0$. Economies are not equally efficient however, and we imagine that an economy is characterized by how much time it takes to produce the per capita output g. Specifically we denote this by $\lambda \geq 1$ where $\lambda = 1$ are countries at the technology frontier, and higher values of λ represent less efficiency in production - due to misallocation, monopoly, corruption, protectionism and other production inefficiencies. Hence $\gamma = g/\lambda$. Our model of battle damage is one in which opponent effort increases proportionally the length of time it takes to produce output. Specifically, we assume that the time to produce γ when there is battle damage is $\lambda + Df_{-k}$. Hence actual per capita output is $g/(\lambda + Df_{-k})$ and expected battle damage is given as

$$d(f_{-k}) = \epsilon\left(\gamma - \frac{g}{\lambda + Df_{-k}}\right) = \epsilon\gamma\frac{1}{\lambda/(Df_{-k}) + 1}.$$

where ϵ is the non-negative common random shock. The shock is assumed to have median equal to one meaning that if the true gain from fighting is zero for a party then there is a $50 - 50$ chance they prefer to fight.

As indicated the realization of the shock is common knowledge at the beginning of the period. It reflects the fact that battle damage is highly random. Indeed wars are often inflicted with random catastrophes, for example, involving the weather. Japan was saved from the overwhelming force of Genghis Khan in 1281 when a divine wind swept away Khan's navy. In 1941, despite complete surprise, poor leadership, complete lack of preparation, and the nervous breakdown of their supreme commander, the Soviet Union was saved from Hitler by the coldest winter in the 20th Century. As a result of this uncertainty and since fighting occur infrequently parties may be optimistic or pessimistic about how great the damage will be. We model this with the shock ϵ representing the degree of pessimism.

The critical feature of the battle damage function is that it is concave, equal to zero at $f_{-k} = 0$ and approaching γ as $f_{-k} \to \infty$. This means that a poorer country at a moment of time, corresponding to a larger λ, has lower marginal battle damage loss from effort. Hence, all other things equal, in the cross-section poorer countries find fighting less costly. By contrast overall economic progress as measured by g impacts all countries the same way, so that increasing income over time will not imply a secular decrease in the propensity for fighting.

4. Main Result

Define the incumbency advantages $\rho_e = B_o/B_i, \rho_f = C_o/C_i$ which are greater than one, the effort cost to damage ratio $r_d = C_0/D$, and the function

$$G(\rho) \equiv \left(\rho^{-1}Q_0 + (1-\rho^{-1})(1-Q_0)\right)\left(\lambda 2 r_d \rho/(1-2Q_0) + V\right).$$

It will later be shown that $G(1)$ is the benefit to cost ratio of fighting for the opposition and $G(\rho_f)$ is that for the incumbent.

Theorem 1. *There are three cases:*

(fighting) $G(1) > \epsilon$: there a fight in which there is probability $\Pi_0 \equiv (1-\rho_f^{-1})Q_0 + \rho_f^{-1}(1/2)$ that the opposition seizes power and the expected cost of effort (relative to GDP) to each of the two parties is the same and equal to $\Pi_0 V$.

(dominant party) $G(1) < \epsilon < G(\rho_f)$: the initial incumbent remains in power.

(voting) $G(\rho_f) < \epsilon$: there is an election in which there is probability $(1-\rho_e^{-1})P_0 + \rho_e^{-1}(1/2)$ that the opposition wins the election.

Comparative statics are given by $dG/d\rho > 0$, $dG/d(\lambda r_d) > 0$, $dG/dV > 0$ and for $\rho < 2$ also $dG/dQ_0 > 0$.

4.1. What Determines Fighting and Voting?

Before turning to the proof of Theorem 1 we examine what it has to say. First it says that whether power is determined by fighting, voting, or belongs to a single dominant party depends (stochastically) only on the four fundamentals of fighting: the productivity-adjusted effort cost to damage ratio λr_d, the value of the prize V (relative to GDP), the incumbent advantage in fighting ρ_f, and the degree of randomness in fighting Q_o. While the fundamentals of elections matter to who wins the election under voting they do not matter in the determination of institutions.

The basic comparative static for fighting and voting from Theorem 1 can be seen from studying the two cutoffs. The first cutoff $G(1)$ measures how attractive fighting is for the opposition and the second $G(\rho_f)$ which measures how attractive fighting is for the incumbent: $dG/d\rho > 0$ implies that fighting is always more attractive to the incumbent than the opposition. When the ratio λr_d is large both cutoffs increase reducing prospects for voting and increasing those for fighting. In particular, all other things equal, countries further behind the technology frontier are both less likely to be democratic and more likely to have fighting. A higher value of the prize V similarly increases both cutoffs reducing prospects of voting and increasing those of fighting. Increases in the incumbent advantage have no effect on the threshold between voting and a dominant party, but increases the cutoff for fighting, meaning greater prospect of fighting. Finally increases the degree of randomness Q_0 unambiguously raises $G(1)$ increasing prospects of fighting. If the incumbent advantage is not too great, it also reduces the prospects of voting.

4.2. On Fighting

Theorem 1 also relates the parameters determining institutions to the nature of fighting if it occurs. The probability that the opposition wins is $\Pi_0 \equiv (1-\rho_f^{-1})Q_0 + \rho_f^{-1}(1/2)$, which is increasing in Q_0 and, since $Q_0 < 1/2$, decreasing in ρ_f. That is, greater incumbent advantage reduces the chances of the opposition winning. Notice, however, that increasing ρ_f (holding fixed r_d) has no effect on the chance that fighting occurs. For example, more repressive state by increasing incumbent advantage reduces the chance that the opposition succeeds in fighting, but does not reduce the chance of fighting unless it also reduces r_d, for example by increasing the amount of damage inflicted on the opposition per given unit of force.

The second result says that the intensity of fighting as measured by the expected cost of effort (relative to GDP) is proportional to the probability of the opposition winning, and the factor of proportionality is exactly the size of the prize (relative to GDP).

5. Proof of the Main Theorem

As players are myopic it suffices to analyze the stage game. The proof of the main theorem then follows from a basic result on contests and some calculations. The basic result on contests is this:

Theorem 2. *Consider a two-party contest with prize V which is won by k with probability given by the increasing convex function strictly convex when strictly increasing $H(g_k/g_{-k})$ with $H(1) = 1/2$, where g_k is effort and the cost of effort is A_k. Define the disadvantaged party d as having $A_d \geq A_{-d}$, and define*

$$\rho = \frac{A_d}{A_{-d}}.$$

Then the disadvantaged party gets utility $u_d = H_0 V$ and wins with probability $\pi_d = H_0 + (1 - 2H_0)(\rho^{-1}/2)$ and the advantaged party gets

$$u_{-d} = \left(1 - H_0 - (1 - 2H_0)\rho^{-1}\right) V.$$

Expected efforts $\overline{G}_k$ are computed from $u_k = \pi_k V - A_k \overline{G}_k$ and in particular the expected cost of effort $A_k \overline{G}_k = (1 - 2H_0)(\rho^{-1}/2)V$ is the same for both parties.

This result is surprising and is neither obvious and nor easy to prove. A slightly weaker version was first shown by Ewerhart (2017) and all of the proofs are based on a crucial idea from Alcalde and Dahm (2007). This particular version follows from Theorem 11.7.1 in Levine, Mattozzi and Modica (2022) based on a similar result in Levine and Mattozzi (2022). We give a brief indication of why it is true. The first idea is that an equilibrium exists. Given this, the second idea is that at the bottom of the support of an opponent's strategy a party faces the expected value of convex utility functions: this must be convex and that means that a party cannot be optimizing at the bottom of the support of the opponent unless they have they same bottom or the bottom is zero. The same support can be ruled out, and the idea extends to show that both must have the bottom at zero. This "bidding down to zero" due to convexity of the contest success function captures Hirshleifer's (1989) intuition about contests and is reminiscent of the ideas in the derivation of equilibrium in the all-pay auction. Notice that both parties will not bid zero in equilibrium with positive probability, so this argument also establishes that the equilibrium must be in mixed strategies.

Bidding down to zero enables us to conclude that one of the parties k gets $H_0 V$ and that the equilibrium utility of the other depends only on the probability that k plays zero. The final idea is to use the method of Alcalde and Dahm (2007) to construct another game in which $-k$ has proportionally higher costs, and k instead of having an atom at zero plays bids with proportionately higher probability. This is an equilibrium of the modified game. Finally, following Ewerhart (2017), we show that this modified game must be symmetric and that this implies that both players must get the same utility, that is $H_0 V$, and have equal probability one-half of winning. Mapping the equilibrium of the modified game back to the original then gives the desired winning probabilities and utilities.

This basic result applies immediately to the election contest. A crucial fact about the fighting contest is that battle damage has no effect on a party's incentives as it depends only on the actions of the other party. Hence the basic result applies also to fighting: we can compute equilibrium without battle damage, then subtracting battle damage from the expected utility of each party. Applying the basic result then yields the following values for the utility, expected effort and probability of winning.

Theorem 3. *Equilibria of the contests satisfy*

	expected effort	*probability of winning*
election incumbent	$(1-2P_0)(\rho_e^{-1}/2)V/B_i$	
election opposition	$(1-2P_0)(\rho_e^{-1}/2)V/B_o$	$P_0+(1-2P_o)(\rho_e^{-1}/2)$
fighting incumbent	$(1-2Q_0)(\rho_f^{-1}/2)V/C_i$	
fighting opposition	$(1-2Q_0)(\rho_f^{-1}/2)V/C_o$	$Q_0+(1-2Q_0)(\rho_f^{-1}/2)$

The main theorem now follows from the result for the stage game:

Theorem 4. *There are three cases:*

(fighting) $G(1)>\epsilon$: elections do not matter, no effort is expended on them, and there is fighting. Define

$$\xi(\rho)=\gamma V\left[\left(1-Q_0-(1-2Q_0)\rho^{-1}\right)\right)-\frac{\epsilon}{(\lambda r_d\rho)/(1/2-Q_0)+V}\right].$$

Then the incumbent party gets $\gamma+\xi(\rho_f)$ and the opposition party gets $\gamma+\xi(1)$ and wins with probability $Q_0+(1-2Q_0)(\rho_f^{-1}/2)$.

(dominant party) $G(1)<\epsilon<G(\rho_f)$: the opposition party concedes and gets nothing while the incumbent gets $\gamma(1+V)$. If an election were to take place and the opposition were to win there would be fighting.

(voting) $G(\rho_f)<\epsilon$: elections take place and the winner takes office. The incumbent party gets $\gamma\left(1-P_0-(1-2P_0)\rho_e^{-1}\right)\right)V$, the opposition party gets $\gamma(1+P_0V)$ and wins with probability $P_0+(1-2P_o)(\rho_e^{-1}/2)$.

Proof. For ease of parsing expressions set $\eta=1/2-Q_0$. From Theorem 3 the expected gain from fighting for the opposition is

$$=\gamma V\left[Q_0-\frac{\epsilon}{\eta^{-1}\lambda r_d+V}\right]=\xi(1)$$

and that of the incumbent is

$$=\gamma V\left[\left(1-Q_0-(1-2Q_0)\rho_f^{-1}\right)-\frac{\epsilon}{\eta^{-1}\lambda r_d\rho_f+V}\right]=\xi(\rho_f).$$

Setting $\xi(\rho)=0$ and solving for ϵ gives the expression for $G(\rho)$.

When the fighting decision is made, the effort expended in the election is a sunk cost. The losing party will fight, then, when the expected utility from the fighting is positive and will not when it is negative. The factor γ is irrelevant. Observe that G is increasing in ρ so if the opposition prefers fighting so does the incumbent and there is a fight and elections are pointless. If the incumbent does not prefer fighting we are in the case of voting, as nobody is willing to fight, the opposition does not concede, the election is credible and no effort is made to overturn the result. In the remaining case the incumbent prefers to fight and the opposition does not, so the opposition concedes rather than commit to a fight it does not want. This is the dominant party case. □

While we consider P_0 equal to zero uninteresting, for completeness we describe the equilibria.

Theorem 5. *If $P_0 = 0$ then in case (voting) there are additional equilibria:.*

(additional voting) $G(\rho_f) < \epsilon$: it is an equilibrium for the opposition to concede with any probability, in which case o gets 0 and i gets γV, and if the opposition does not concede, the outcome is as described in case (voting) above. Conceding with probability 1 Pareto dominates all other equilibria.

6. Empirical Approach

The key conclusion of the model is that the same economic fundamentals that explain the prevalence of fighting also explain the prevalence of democracy. We empirically examine whether this is true, and assess the impact of income on voting and fighting in several stages.

First, we consider the conflict resolution function and estimate the chances of success against overwhelming odds, Q_0, and the incumbent advantage, ρ_f. Here we use only data on the outcomes of civil wars and the (military) forces of the incumbent and opposition in these wars.

In the second stage we assess the impact of income on fighting. Here we rely on data on GDP per capita and the incidence of fighting. We first estimate the relationship between productive inefficiency λ and the probability of fighting. We then turn to the implications the cost to damage ratio, r_d, and the size of the prize, V. From this, we then estimate the distribution of the shock ϵ as having two uniformly distributed segments.

Using the estimated parameters from the fighting data we are able to predict how likely countries are to award power through voting. While this prediction is relatively accurate, it modestly understates the probability of voting in countries that are very productively inefficient. As these estimates were extrapolated from estimates of the distribution of optimistic shocks, we allow for a third uniform segment of the distribution of ϵ for pessimistic shocks and show that this gets the probability of voting in productively inefficient countries right.

Subsequently we use data on oil and natural gas production to examine the impact the size of the prize, V, on the occurrence of voting. We examine as well the role of ethnic division. The data, explained in more detail below, spans from 1815 to the present. The theory helps inform us about the appropriate length of a period. The time between major elections in the frontier countries, the US and the UK, are about five years. This suggests that a parties may reasonably expect to hold power for about this length of time before a new contest takes place. Hence we aggregate our data into half decade averages. The periods of World War I and World War II were excluded as the model does not apply to countries involved in heavy warfare with other countries. In all there are 4160 observations on 215 countries. Summary statistics and sources for the variables discussed below are in Table 1.

6.1. The Data

6.1.1. Fighting

The data on civil wars is from the Correlates of War (COW) project on civil conflicts as described in Dixon and Sarkees (2016). We use the intra-state wars dataset, and focus on wars coded as a civil war over central control as this is the type of fighting contemplated in the model. We construct three variables: the ratio of the effort of the weaker side to the stronger side (denoted byf_k/f_{-k}) , the ratio of the expected effort of the incumbent to that of the opposition (Ef_i/Ef_o) and a binary variable indicating which side won.

As a proxy for effort we take the maximum in theater forces for that party during the conflict. If this data was missing for either party the observation was excluded. However, person power is not all equal, and some soldiers are more effective than others. In particular the incumbent will generally have better trained and equipped forces, better bases and so forth. Based on Dupuy's (1986) estimate that in 1941 German troops were about three times as effective as Russian troops, we took incumbent effort f_i to be triple the number of in theater forces while for the opposition f_o we took it to be the number of in theater forces.

The outcome variable is binary being equal to one if the weaker side won. If both sides had equal strength this was scored as an 0.5 chance of the either side winning. If neither side won then the state remained in control so this is classed as a win for the state actor, that is the incumbent. To be consistent with our five year period length, if the civil war lasted more than five years we broke it into five year periods with the state actor winning until the five year period in which the war ended.

6.1.2. Fighting

A fighting outcome should include substantial unrest designed to bring down the government, or at least consistent with that possibility. This would include general strikes and widespread protests as well as attempted or successful coup d'etats. Unfortunately we observe only the latter. To get the most comprehensive possible dataset, we augment the data on civil conflict from the COW project with data on coups from the Wikipedia list of coups and coup-attempts by country.[2] These two datasets combined give us a overview of incidences of civil unrest, or fighting, between 1815 to the present, in five year periods.

We assume a period to be a fighting period if there is fighting at any point during the period. If there is no fighting, the period is classified according to whether or not there is a dominant party or there is voting, as described below. A fight is said to occur in a five year period if there is a civil war or the continuation of a civil war lasting more than five years, a coup, or a coup attempt. While the COW dataset records the duration of most conflicts, this data is not available for all coups; when missing the duration of a conflict event is set to one year. In our data, there are 561 cases of fighting: 151 civil wars, 244 coups and 144 attempted coups.

[2] See: https://en.wikipedia.org/wiki/List_of_coups_and_coup_attempts

6.1.3. Income

Population and GDP data are from the Maddison Project as described in Bolt and van Zanden (2020), and supplemented with data from the World Bank. GDP per capita, referred to as income, is expressed in 2011 US dollars.[3] Income determines the *ex post* empirical value of current GDP relative to the frontier $\tilde{\lambda}$. We refer to this as contemporaneous productive inefficiency. In computing $\tilde{\lambda}$, the frontier country is always taken to be the country with the highest per capita GDP and with at least 0.5% of world population. This excludes small countries with a lot of mineral wealth (the Middle East) and countries with large banking sectors (Ireland, Luxembourg, and so forth). The frontier country is the UK until 1880, when it switches to the USA. This is consistent with the history of technology. Countries with per capita GDP higher than the frontier country are assumed to have $\tilde{\lambda} = 1$, that is, to be on the frontier.

Contemporaneous productive inefficiency must be distinguished from our explanatory variable λ, which is the *ex ante* anticipated value of $\tilde{\lambda}$, which we refer to simply as productive inefficiency. These *ex ante* beliefs, λ, by the parties about the ratio determine their beliefs about battle damage. The natural measure of λ is to take $\tilde{\lambda}$ from the previous period, but we must measure what the ratio "would have been in the absence of fighting." In particular we would like to account for the fact that if there was heavy fighting in the previous period $\tilde{\lambda}$ may be artificially depressed.

Our base assumption is that recovery from a fight is fast: that recovery from battle damage is swift—on the order of five years—which is supported by data from World War II. For example, the population of Hiroshima in 1940 was 1.9 million and in 1950—five years after the nuclear bombing—it was 2.1 million.[4] Figure 4 below shows the impact of three major wars on productive inefficiency: the impact of World War II on Germany and France, and the impact of the US Civil War. In both Germany and France full recovery occurs about ten years (two of our periods) after the peak damage, and after five years more than half the damage is gone. Note that the initial strong recovery is unlikely to be due to the Marshall Plan which was passed into law in the USA only in April of 1948. In the USA the Civil War seems to have had no impact on productive efficiency at all.

[3]There is a slight difference between the Maddison data and the World Bank data where they overlap. We corrected this by using the ratio of US values in 1960.

[4]http://www.demographia.com/db-japanpref.htm

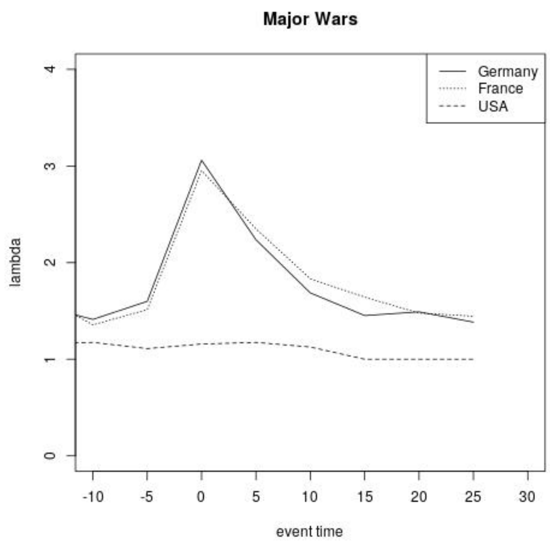

Figure 4: Productive Inefficiency Due to Major Wars

World War II was particularly intense compared to civil wars, even the US Civil War. In World War II the major combatants were Germany and the USSR. The death rate per 100,000 population per year for those two countries combined was about $3,000$.[5] By contrast, Levine and Modica (2018) collected similar data for post World War II civil wars. The most intense wars were in the Sudan, Syria, and Lebanon, where the corresponding death rates were 330, 380 and 400 respectively: nearly an order of magnitude less than during World War II.

Based on the fact that even after very intense fighting full recovery takes about ten years, it makes sense to think that in the less intense fighting of a civil war full recovery takes about five years. This means that anticipated productive inefficiency should be what is was prior to the outbreak of fighting. However, fighting does not depress $\tilde{\lambda}$ in all cases (for example a coup is unlikely to have much effect). Hence we take our explanatory variable λ to be the smaller (more efficient) value of last period $\tilde{\lambda}$ and the last value of $\tilde{\lambda}$ before fighting broke out. For the sake of brevity we refer to this as the "lagged GDP ratio".

A final issue is whether, despite their limited intensity, very long civil wars might have a cumulative effect on productive inefficiency. To check for this we examined data from Sudan which had two very lengthy civil wars. This is shown below in Figure 5 with the shaded areas corresponding to the civil war periods.

[5] Based on data from Wikipedia article on World War II casualties. The war between Germany and the USSR lasted about four years and we attributed all German casualties to those four years.

For comparative purposes the data for a neighboring country, Kenya, which had no civil wars is shown.

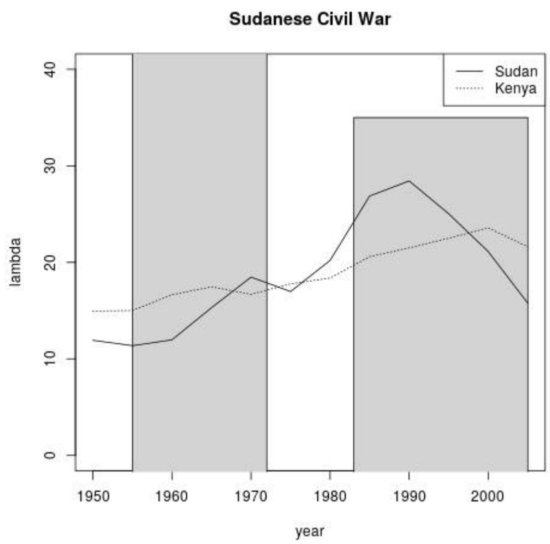

Figure 5: Productive Inefficiency in Sudan

As can be seen there is not much evidence of a cumulative effect. It is true that productive inefficiency rose substantially from prior to the first civil war to the middle of the second: but most of that took place during the period between the civil wars, and in the second civil war productive inefficiency dropped substantially.

6.1.4. Voting

Data on democracy is from the Varieties of Democracy (V-Dem) project as described in Pemstein et al. (2024). Our basic measure of voting is based on the polyarchy index, which captures the extent to which electoral democracy is achieved in a country. It is based on measures of freedom of association and expression, cleaness of election, the extent suffrage, and whether or not country leaders are elected. This does not measure exactly what we want, as it places emphasis on the extent of the franchise which is not part of our theory. From about 1960, the franchise, when there is voting, is generally universal and the index, which has been carefully constructed and well tested seems to do a good job in measuring voting.

To use the index, which ranges from zero to one, we need a cutoff indicating when voting takes place and when it does not. Most intermediate level cutoffs do a good job picking out countries that are clearly democracies such as the US and Western Europe, as well as countries that are clearly not such as China and the Gulf States. Most intermediate cutoffs also perform well for countries where there was a large institutional change that affected voting such as Argentina, South Korea, Mexico and Taiwan.

However, to use the index to measure voting correctly, we need it to capture poor countries that may hover on the edge of voting. We did this by examining Benin as a case study. Benin is a poor country that has had a brief episode of democracy. The polyarchy index for Benin is shown below in Figure 6.

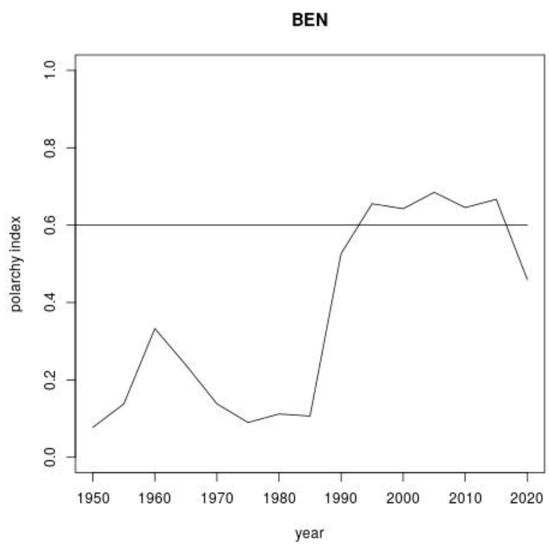

Figure 6: Polyarchy in Benin

From Wikipedia the salient facts about voting in Benin are these. Benin (then the Republic of Dahomey) achieved independence from France in 1960. There followed a period of civil strife culminating in the seizure of power by Lt. Col. Mathieu Kérékou, who renamed the country as the People's Republic of Benin. Dominant party rule ensued until the inability to pay the army and a banking collapse resulted in an agreement for a new constitution and the renaming of the country as the Republic of Benin. An election was held in 1991, and the incumbent Kérékou lost to opposition leader Nicéphore Soglo who took power. There ensued a number of elections in which the winner was often not the incumbent and power was transferred peacefully. This lasted until Patrice Talon was elected in 2016. Talon then pushed through electoral reforms that disenfranchised the opposition and put many opposition leaders in prison.

From our perspective, power in Benin was established by voting in roughly the period from 1991 to 2016: this is reflected in the fact that the polyarchy index in Figure 6 is considerably higher in those years than in other years. Taking a cutoff of 0.6, the horizontal line in figure 6 seems to capture the period of voting well. We take this as our basic cutoff.

However, as indicated, there is a major issue with the polyarchy index before 1960 that needs to be accounted for: the polyarchy index places substantial weight on the extent of the franchise and a measure of free fair elections. Our definition of voting, however, is that the loser should respect the outcome. While the franchise is important for democracy in the usual sense, it is not so relevant for voting in our case. For example, the disenfranchisement of women is undemocratic, but there is not a "men's" party and a "women's" party that take turns in power. Hence, while the polyarchy index seems to a good job of measuring voting in the cross-section it does poorly over time: the UK had peaceful transitions of power based on voting in the entire period and the US as well, except briefly around the time of the Civil War.

There are two methods of accounting for voting provides the measure we are seeking prior to 1960. One is to revise the index by reweighting the components. We experimented with this, but found that it is all too easy to get meaningless results. Instead of replacing a well thought out and well tested index with our own, we decided instead to adjust the cutoff for democracy. A simple and useful way of doing this is to use the value of the polyarchy index of the frontier country as an indicator of the proper standard. Specifically, we chose the cutoff to be 95% of the polyarchy index in the frontier country or 0.6—whichever is smaller. This cutoff is shown below in Figure 7 below.

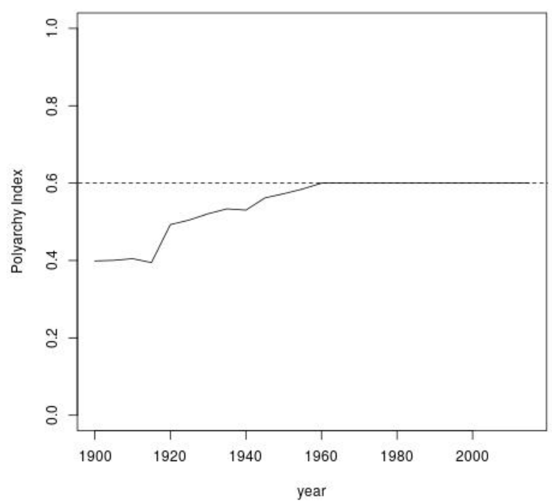

Figure 7: Polyarchy Index for the Frontier Country

6.1.5. Oil and Gas

The per capita value of oil and natural gas is from Ross and Mahdavi (2015) and extends through the five year period ending in 2009. We adjusted the year 2000 US dollars to 2011 US dollars using GDP deflator from the World Bank.

Variable	Mean	SD	Observations
Weaker to stronger f_k/f_{-k}	0.29	0.24	123
Incumbent to opposition Ef_i/Ef_o	3.96	N/A	123
Weaker side wins	0.28	0.45	123
Fighting	0.13	0.34	4160
No voting	0.74	0.44	3442
$\tilde{\lambda}$	8.95	10.91	3442
λ	8.63	10.24	3442
GDP/capita	9761	11514	1641
Oil & Gas/capita	918	4241	1641
Ethnic Division	0.044	0.048	1368

Table 1: Descriptive statistics

6.1.6. Ethnic Division

Our measure of ethnic division is the polarization index from Esteban, Mayoral and Ray (2012). This captures the distances between groups in a country based on similarity between languages and the population share of the groups. This index (PEthnoDelta005) is available for 1960 to 2008.

6.1.7. Data Summary

The variables used in the study and summary statistics are in Table 1 below.

7. The Conflict Resolution Function and Q_0

We start by using data on the outcome of civil wars to assess the conflict resolution function. Recall that if k is the lesser effort of the two parties the conflict resolution function $Q(f_k/f_{-k})$ should depend upon relative effort and be weakly increasing and weakly convex. Moreover, as $f_k/f_{-k} \to 0$ it should converge to $Q_0 > 0$. Is this true, and what is Q_0? To summarize the findings of this section, we find that the assumption of the probability of success depending only on relative effort and being weakly decreasing and weakly convex is consistent with the data. We estimate that Q_0, the chances of success against overwhelming odds, is about 7% and that incumbent advantage is about four to one.

7.1. The Conflict Resolution Function

Since convexity is an issue and there are non-integer values of the endogenous variable, we used a linear probability model when assessing the conflict resolution function. We studied a generalized conflict resolution function of the form $\tilde{Q}(f_k/f_{-k}, f_{-k})$ and approximated this by a quadratic. According to the model, the shock determining who wins is independent of the decision to fight and the decision about the size of forces to commit. And although the right hand side variables are endogenous, they are predetermined and OLS yields consistent estimates.[6] The results are reported in Table 2. None of the fitted values exceeded one, and only three were negative being equal to −0.013. Note that except for the constant term, none of the coefficients are estimated with much precision.

	Quadratic		Force ratio only	
Variable	Estimate	SE	Estimate	SE
Constant	0.18	0.11	0.14	0.06
f_k/f_{-k}	0.94	0.61	0.49	0.16
$(f_k/f_{-k})^2$	−0.64	0.68		
f_{-k}	−1.02	0.76		
$(f_{-k})^2$	1.23	2.09		
$(f_k/f_{-k})f_{-k}$	−2.04	0.90		

Table 2: Conflict Resolution: Probability Weaker Party Wins

We are interested in whether the conflict resolution function depends only on the force ratio and is a convex function. As the coefficient on the quadratic term in the force ratio is negative, indicating a possible failure of our convexity assumption, we reran the regression to examine how the model linear in the force ratio only fares.[7] The results are in column four of Table 2. None of the fitted values are negative. The intercept term 0.14 is an estimate of Q_0, the chance of success against overwhelming odds. Moreover, by omitting the four variables the sum of squared residuals increases by 5.2%. Multiplying this by the sample size, under the null hypothesis, this result approximately follows a chi-square distribution with four degrees of freedom. The probability of getting 5.2% or larger is 16%, well above standard criteria for statistical significance. In summary: although the evidence is weak, it is consistent with our theoretical assumptions that only the force ratio matters and that the probability of success is convex in the force ratio.

7.2. Measurement Error

As we used a proxy for effort there is a potential problem of measurement error. As the estimate of the constant term is negatively correlated with the estimate of the slope term, the measurement error in the force ratio will bias the constant term up leading to an overestimate of Q_0 (see, for example, Levine (1985)). To account for this potential bias, we employed a robust technique for estimating Q_0 that is consistent in the face of measurement error.

We compute for values of the proxy inverse force ratio $\varphi \equiv f_{-k}/f_k$ that exceed a threshold $\overline{\varphi}$ the percentage of the times that the weaker party wins. In

[6] This would not be true in a model of asymmetric information in which fighting or force commitments depend on private information about who is likely to win.

[7] Note that linearity in the force ratio is not strictly convex, but if we cannot reject linearity we cannot either reject a slight among of strict convexity.

Online Appendix 1 we give conditions under which it is possible to choose $\overline{\varphi}$ as a function of the sample size so that asymptotically this converges in probability to Q_0. The idea is that if we observed the true inverse force ratio $\tilde{\varphi}$, then the probability of lying above a threshold should asymptote to Q_0. Hence the same should be true for estimates based on the proxy force ratio.

The results of the estimation are graphed in Figure 8 below.

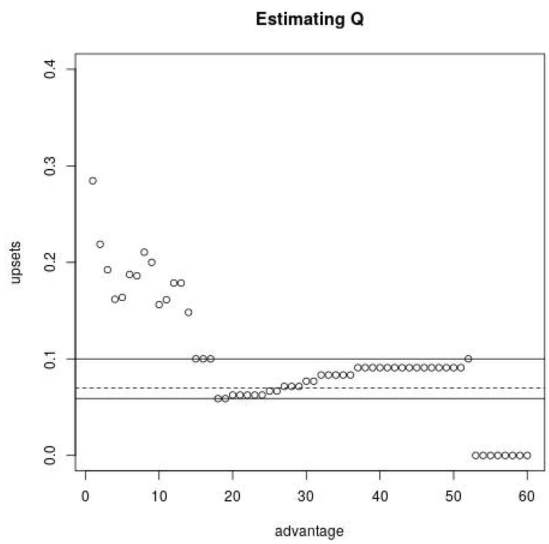

Figure 8: Force Ratios and Upsets

Naturally, if we take $\overline{\varphi}$ too large there are too few observations to get a good estimate of Q_0. In Figure 8 it can be seen that this happens at $\overline{\varphi} = 53$, where the number of observations has fallen from 10 to 5 . For lower force ratios we see that as the advantage increases, the probability of the weaker party winning stabilizes in the range from 6% to 10% (indicated by the solid lines in Figure 8). This is less than the linear probability model estimate of 14%. As there are more data points in the lower range for force ratios around 30, we take 7% as a plausible value of Q_0. This is the dashed line in Figure 8.

7.3. Incumbent Advantage

Finally, we turn to estimating incumbent advantage, $\rho_f = C_o/C_i$. From Theorem 3 equilibrium expected effort is given by $Ef_k = (1-2Q_0)(\rho_f^{-1}/2)V/C_k$ from which

$$\rho_f = \frac{C_o}{C_i} = \frac{(1-2Q_0)(\rho_f^{-1}/2)V/C_i}{(1-2Q_0)(\rho_f^{-1}/2)V/C_o} = \frac{Ef_i}{Ef_o}.$$

Using the civil war data, we compute for each observation the forces of the incumbent and opponent as measured in numbers per capita. Taking the average over the sample, we find for the incumbent $Ef_i = 1.21\%$ and for the opposition $Ef_o = 0.31\%$, which gives the estimate $\rho_f = 3.96$.

8. GDP and Fighting

Our estimation strategy is to assume that the only difference between countries lies in their time to produce, captured by λ. We have already used data on who won civil wars to estimate the chance of winning against overwhelming odds (Q_0) and the data on force ratios to estimate (ρ_f). We now use data on GDP per capita and the incidence of fighting to estimate the distribution of the shock ϵ, r_d and V. In the next section we ask whether these estimates can also explain the incidence of voting.

We find that the size of the prize V matters little for the chances of fighting, but that there is a strong relationship between λ and the chances of fighting. This relationship is well-described by a continuous piecewise linear function with an initially steeply upward sloping segment followed by a relatively flat segment. Near the frontier, where $\lambda = 1$, the probability of fighting is small at about 4%. For countries with 20% of frontier income this probability rises to nearly 14%. For very poor countries with less than 6.5% of frontier GDP, this rises even somewhat higher to 22%. We also find that fighting does not generally pass a benefit cost test but occurs when there are optimistic beliefs that battle damages are small.

8.1. Fighting and λ

Let define $\Xi(x)$ to be the cdf of ϵ. According to the Theorem 1 the probability of fighting is given by $\Xi\left(Q_0\left(\lambda 2r_d/(1-2Q_0)+V\right)\right)$. Here the distribution itself, along with r_d and V, is unknown, while Q_0 is known from the estimation in Section 7. Our first step is to estimate the relationship between λ and the probability of fighting, and to then use this to assess the structural parameters. This will enable us to determine a relationship between r_d and V. We then estimate V and in turn use this to get a final estimate of r_d.

In order to assess the relationship between λ and the probability of fighting, we first group observations into categories k with cutoff points for λ of the form $(1.25)(1.75)^{k-1}$, and compute the probability of fighting for each category. The maximum value of λ in the data is 110. Table 3 below reports the mean value of λ for each category, together with the estimated probability of fighting, the standard error for the binomial average and the number of observations.

λ	Prob. of fighting	SE	Observations
1.08	0.043	0.012	281
1.67	0.085	0.011	634
3.00	0.112	0.012	723
5.05	0.139	0.011	1054
8.58	0.153	0.014	674
15.79	0.196	0.019	420
26.59	0.220	0.025	264
47.74	0.227	0.040	110

Table 3: GDP and the Probability of Fighting

As indicated by the standard errors there is not much issue with sampling error. Below in Figure 9 we plot λ against the probability of fighting (marked by circles). Note that there is data only for the lower portion of the graph in this figure and Figure 11: there are very few observations of countries with λ greater than 50. The region of higher λ is shown in order to show the implication of the theory for relatively poor countries.

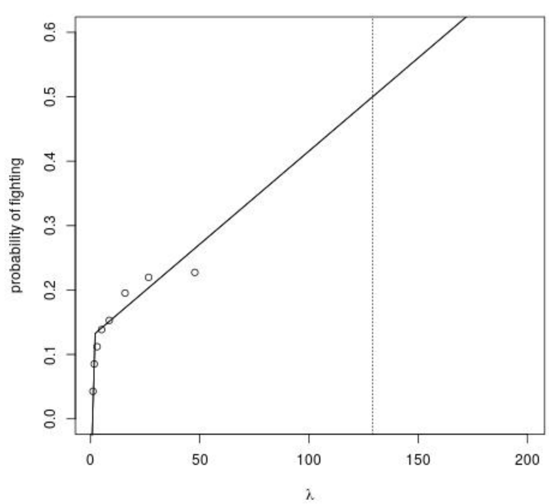

Figure 9: GDP and the Probability of fighting
Dots: fighting data aggregated in bins shown in Table 3. Solid curve: estimated probability of fighting. Vertical line: median value of λ.

To a good approximation this is a continuous piecewise linear function with two linear segments. Hence we fitted a segmented linear probability model, and choose the cutpoint by minimizing the sum of squared residuals as is standard (see, for example, Feder (1975)). The estimating equation is

$$Ey_\tau = \begin{cases} \alpha + \beta_r \lambda_\tau & \lambda_\tau < \lambda_c \\ \alpha + \beta_p \lambda_\tau & \lambda_\tau \geq \lambda_c \end{cases}$$

where τ indexes time and country and y_τ takes on the value one if there is fighting and zero otherwise. The results are below in Table 4 and plotted as the solid line in Figure 9.

			Prior to 1955	
	Coefficient	SE	Coefficient	SE
α	0.133	0.008	0.126	0.012
β_r	0.116	0.036	0.094	0.060
β_p	0.003	0.0005	−0.002	0.003
λ_c	2.09	0.245	2.05	0.417
Observations	4160		1263	

Table 4: Fighting Estimation

The cutpoint is estimated to lie at 2.09 with corresponding probability 0.13. The function is estimated to be 0.006 at $\lambda = 1$ and reach 0.5 at $\lambda = 129$, corresponding to the median for $\epsilon = 1$. This is marked with a vertical dotted line.

In Figure 9 the top bin with average $\lambda = 47.74$ has a probability of 0.227 which lies below the estimated line where the fitted probability is 0.265. However, there are only 110 observations in the top bin and the standard error is 0.040 so that the discrepancy is less than one standard deviation, and hence consistent with sampling error.

8.1.1. Structural Stability

Starting in around 1955 there are enormous changes due to former colonies becoming independent. To check for structural stability we estimated the model using only data from 1950 and earlier The results are also shown in Table 4. The coefficient β_p for the upper region, the poor countries, is negative in this earlier sample. However, the estimate is very imprecise and the estimate for the full sample is less than two standard deviations away. The imprecision is due to the fact that there are very few observations of poor countries in the early sample: only 28 observations with $\lambda \geq 12$. The remaining coefficients in the early sample are quite similar to those in the full sample.[8]

This structural stability has important implications for the relationship between absolute GDP per capita and fighting that we examine below in section 11.

[8] We also estimated the model using only data from the prewar period 1935 and earlier. This yields similar coefficient estimates.

8.2. Estimating r_d and V

Having estimated the relationship between λ and the probability of fighting, we turn to the implications for r_d and V. At $\overline{\lambda} = 129$ there is a 50% chance of fighting, so this is the median. Since the median of ϵ is assume to be one, this means that $Q_0\left(\overline{\lambda}2r_d/(1-2Q_0)+V\right)=1$. We have already estimated Q_0 to be approximately 0.07. This enables us to establish the relationship between r_d, the cost to damage ratio and V, the size of the prize:

$$r_d = \left(\frac{1-2Q_0}{2\overline{\lambda}}\right)\left(\frac{1}{Q_0}-V\right) = 0.0033\,(14.3-V)\,.$$

Since $0 \leq V \leq 1$, this gives a fairly tight bound on $0.044 \leq r_d \leq 0.048$. This says that paying \$100 results in damage of about \$2,000.

Using the piecewise linear function, we see that in the upper region for low income countries

$$\begin{aligned}&\Xi\left(Q_0\left(\lambda 2r_d/(1-2Q_0)+V\right)\right)\\ &=\Xi\left(0.077\lambda+0.07V\right)\approx 0.124+0.0033\lambda,\end{aligned}$$

from which we see that the slope of the cdf in this upper range is Ξ' = 0.38. Hence the derivative of the probability of fighting with respect to V is 0.026, meaning that an increase in the size of the prize from 0% of GDP to 100% of GDP would increase the chances of fighting in poor countries by only 2.6%. Note that this is an empirical result: the theory does not force low sensitivity to V, and indeed for rich countries the sensitivity is much greater.

8.3. Implications for V

One implication of this analysis is the the chances of fighting in poor countries are not all that sensitive to V. However, the intensity of conflict as measured by the cost for each party is $(1-2Q_0)(\rho_f^{-1}/2)V$, which is highly sensitive to V. In other words, the chances of fighting depend on a cost benefit analysis and increasing the size of the prize increases the benefits. Simultaneously, an increasing size of the prize increases the cost since the cost is endogenous, and both parties will incur a greater cost to get a greater prize. Hence there is not so much sensitivity in the cost benefit analysis about whether to engage in civil war. By contrast, if a civil war does start, the intensity of the fighting is directly proportional to the size of the prize. Earlier research did not clearly make this distinction.

Although it matters little in analyzing fighting, the size of the prize does matter for the analysis of voting. One measure of V is discretionary spending in a country with an advanced tax system. In the modern frontier country, the US, for the period 2003-2022, discretionary spending averaged 7.3% of GDP.[9] We take $V = 0.073$, leading to a corresponding $r_d = 0.047$.

[9] Congressional Budget Office Discretionary Spending in Fiscal Year 2023: An Infographic.

8.4. Implications for ϵ

Overall fighting is driven by optimistic draws of ϵ. Recall that the median of ϵ is one, and that in this case when the true benefit cost ratio is one there is a 50% of an optimistic shock resulting in a civil war. In fact, no country is so poor that the true benefit cost ratio is greater than or equal to one. In other words, civil wars take place when the true benefit cost ratio is less than one—a civil war has an expected loss—but beliefs about battle damage are optimistic, i.e., that battle damage is low.

Knowing $r_d = 0.047$ and $V = 0.073$, we can find the implied distribution of ϵ. The bottom of the support of the cdf Ξ is 1.2%, meaning that the most optimistic belief is that battle damages are such that they are very small. Low values of ϵ below the cutpoint of 2.1% have a relatively high density with the probability of being at or below the cutpoint being 13%. Above this, the density is lower with the cumulative probability rising to 50% at the median of 100%. As there are no observations above the median there is no information about the distribution of ϵ above this point.

9. Implications for Voting

Having estimated all the relevant parameters, we can now make a prediction about how likely countries are to award power through voting. This is show in Figure 10 below where the data on democracy is aggregated using the same cells as used in the fighting data.

While this does quite well, as can be seen it predicts no voting with probability one for very inefficient countries, while in fact the probability of no voting is more like 90%. However, the fighting data does not tell us about the distribution of ϵ above the median, and it is this distribution that determines how likely voting is in the higher range of λ. With this in mind we estimated the upper part of the curve using the voting data.

The result using the voting data for the higher values of λ is shown in Figure 11 below. For comparative purposes the lower solid curve is the estimated probability of fighting, the same as in Figure 9. The upper curve is the probability that there is no voting: that there is either a dominant party or fighting. As indicated, the solid part of this curve is derived from the theory and the estimates from the fighting data. The upper dashed part of the curve representing the cdf of ϵ above the median is estimated from the voting data. The dots represent the actual probabilities using the same bins as in Table 3. The result is striking: the theory fits the data extremely well.

Notice that for poor countries the chances of dominant party rule actually decline with distance to the frontier, since the probability of fighting rises faster than that the chances of voting decline. For middle income countries the opposite is true,with the probability of fighting rising slowly but the chances of voting declining rapidly. Note that this is exactly what happens in the data.

For future reference note that the probability of voting becomes 50% at $\lambda = 2.05$.

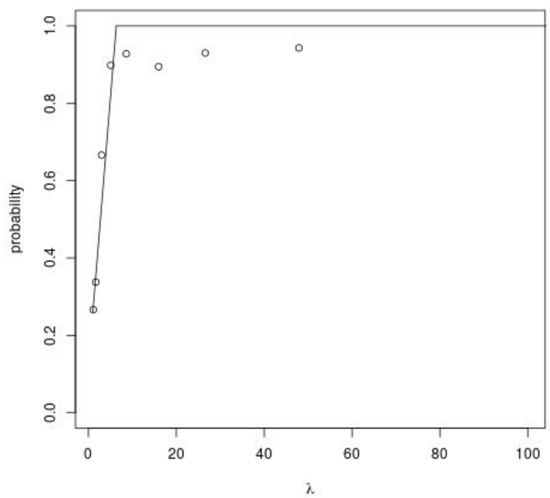

Figure 10: Probability of no voting estimated from fighting data
Dots: voting data aggregated in bins shown in Table 3. Solid curve: predicted values from fighting estimates.

9.1. Finding Voting

We turn now to the calculations underlying Figure 11. Recall that the probability that there is no voting (either fighting or dominant party) is given from Theorem 1 as

$$\Xi\left[\left(\rho_f^{-1}Q_0+(1-\rho_f^{-1})(1-Q_0)\right)(\lambda 2r_d\rho_f/(1-2Q_0)+V)\right].$$

Using the estimates $Q_0 = 0.07$, $\rho_f = 3.96$, $V = 0.073$ and $r_d = 0.047$ this becomes

$$\Xi\left[0.71\,(0.43\lambda+0.073)\right].$$

From this we can compute that the cutpoint for no voting is negative, so that only the upper segment is relevant. The corresponding probability of no voting in terms of λ is then $0.14+0.12\lambda$. This is the solid upper curve plotted in Figure 11.

9.2. Voting and Income

As λ increases, the probability of no voting approaches 1 and we have no fighting data for the segment above the median which for voting is about 3.1. The data on no voting, indicated by dots using the same bins as for fighting and shown in Figure 11, indicates that the probability can be well approximated by two linear segments: we again implement a segmented regression. We hold fixed the curve estimated from the fighting data $(0.14+0.12\lambda)$ up to a cutpoint, and

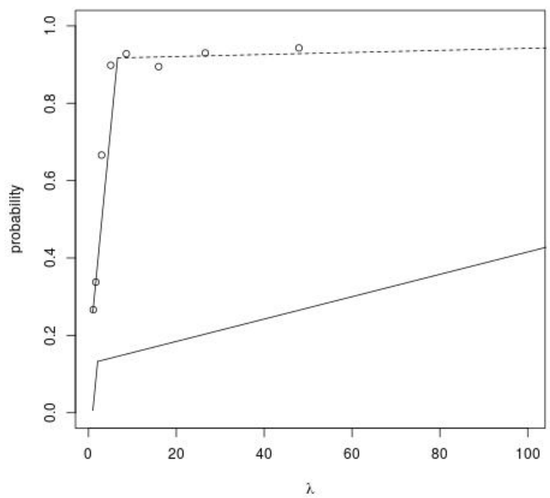

Figure 11: fighting (lower line) and no voting (upper line)
Dots: voting data aggregated in bins shown in Table 3. Lower solid curve: estimated fighting probability. Upper solid curve: estimated voting probability based only on fighting data. Upper dotted curve: estimated voting probability based on voting data.

used the no voting data above the cutpoint. The results are shown in Table 5 below and the no vote data is represented by the dashed line in Figure 11.

	Coefficient	SE
α	0.144	N/A
β_r	0.117	N/A
β_p	0.003	0.0009
λ_c	6.63	0.06
Observations	3442	

Table 5: Non-voting estimation

The data is also summarized in Table 6 below.

λ	Predicted	Actual	SE	Observations
1.09	0.272	0.266	0.030	214
1.67	0.339	0.338	0.020	542
2.98	**0.492**	**0.666**	0.020	545
5.06	**0.735**	**0.898**	0.011	916
8.62	0.920	0.928	0.011	582
15.97	0.921	0.895	0.016	361
26.59	0.924	0.930	0.016	257
47.91	0.930	0.942	0.023	105

Table 6: GDP and the Probability of No Voting

Vertical distances are hard to read from the graph in Figure 11. Except for the highlighted values, the predicted and actual values are quite similar, and well within the standard errors of the estimates. The exceptions are 2.98 and 5.06, where the theory under predicts the data by about 16%. We have enough data that our model—an approximation to the truth—can be rejected by standard statistical tests.

Indeed: the data favors shifting the democracy curve slightly to the left. This poses a problem when we want to use voting data to assess V on subsamples, that is to estimate equations of the form $Ey_{\ell\tau} = g_\ell(V, \lambda_\tau)$ where $\ell \in \{\text{fight}, \text{novote}\}$. If we estimate this equation on the full sample using the voting data we get the results shown in Table 7 below.

	Vote
V	0.495
SE	0.031
Observations	3422

Table 7: Variable V

As can be seen this leads to a wild overestimate of V, confirming that our approximate model is not "true." Note that if we do the same estimation for the fighting data we naturally recover the value $V = 0.073$. To use the voting data on subsamples, we must avoid the overfitting that takes place on the full sample. We do so by adding the difference between the estimated V and 0.073 and estimating $Ey_{\text{novote},\tau} = g_{\text{novote}}(V + (0.495 - 0.073), \lambda_\tau)$ which will result in the proper result of 0.073 when run on the full sample.

10. Income and Democracy

It could be of course that nations are rich because they are democratic, rather than, as in our theory, that they engage in voting because they are rich. There is indeed a long-standing debate in the literature over this. Lipset (1959) argued that democracy is persistent only in rich countries, and this has found support in cross-country regressions such as those of Barro (1999). Acemoglu et al.

(2008) argue that this correlation is spurious. Although the statistical methods used are controversial (see Che et al. (2013)) here we accept the procedure of Acemoglu et al. (2008) and show that their evidence that there is "no causal effect of income on democracy" is in fact consistent with our theory that there is.

10.1. Cross-Sectional Evidence

Acemoglu et al. (2008) show that when a measure of democracy is regressed on income and time and country fixed effects are included, the correlation between democracy and income is either insignificant or negative depending on the estimation technique. To assess this argument we ask what the Acemoglu et al. (2008) procedure will find if the data is generated by our model.

We generate an artificial dataset parallel to that used by Acemoglu et al. (2008). We take our five year data for their sample period starting in 1955 through 2000, omitting countries for which data on GDP per capita is missing. We then apply our theory to compute the probability of voting ν_τ : taking the relevant value of λ_τ this is given by

$$E\left[1-\nu_\tau\right] = \begin{cases} 0.144 + 0.116\lambda_\tau & \lambda_\tau < 6.63 \\ 0.912 + 0.000266_\tau\lambda_\tau & \lambda_\tau \geq 6.63 \end{cases} .$$

We presume that ν is what the different democracy indices are trying to measure so take it as our measure of democracy. Letting z denote the logarithm of GDP per capita, we then use OLS to estimate the same equation estimated by Acemoglu et al. (2008): $\nu_{it} = \alpha\nu_{it-1} + \gamma z_{it-1} + \mu_t + \delta_i$. Below in Table 8 is the result of that estimation along with the findings of Acemoglu et al. (2008):

	Theory	Acemoglu et al. (2008)	
		OLS	GMM
α	0.429(0.029)	0.379(0.051)	0.489((0.085)
γ	−0.055(0.004)	0.010(0.035)	−0.129(0.076)

Table 8: Theory versus Data (standard errors in parentheses)

Acemoglu et al. (2008) found that when they did OLS using time and country fixed effects the coefficient on z is small and insignificant. When they use more advanced estimation techniques—generalized method of moments (GMM)—the point estimate in fact becomes negative. If our model is true, and indeed income does cause democracy, in fact the coefficient on z should be negative. Not only that, but the coefficients on both the lag of democracy and the lag on z estimated from our model are quantitatively similar to those coefficients found by Acemoglu et al. (2008). The bottom line is that the Acemoglu et al. (2008) procedure yields misleading results: it suggests that lagged democracy is important and lagged GDP is not, despite the fact that the data is generated without any persistence in democracy and is determined only by income.

To get an intuition into what is going on, observe first that the level of GDP is different than the distance to the frontier, but that this does not matter in the presence of time fixed effects since these can be made equal to the log of the lagged GDP ratio. However, while the true relationship (even in logs) is highly non-linear (rising rapidly initially and then becoming extremely flat) the Acemoglu et al. (2008) model supposes that the relationship (in logs) is linear.

Suppose, for the purpose of understanding, that nothing changes over time so we are just observing countries some with high and some with low λ. Since the relationship is highly non-linear, using z_{it-1} on the right-hand-side leads to a lot of error. On the other hand, country fixed effects can perfectly account for the non-linear relationship, so the coefficient on λ_{it-1} will be zero and the fit will be perfect. In other words: the only way in which the Acemoglu et al. (2008) model can accommodate the non-linearity of the data is through the country fixed effects, and it is this specification error that leads to misleading results.

When we account for time this makes things even worse. Poor countries have substantial fluctuations in income over time but because ν is very flat as a function of λ, these income fluctuations result in very little change in ν. Hence putting weight on ν leads to substantial error and the regression will avoid doing this.

10.2. Does Democracy Cause Income?

The attempt to debunk the idea that income leads to democracy is in part an effort to support the point of view that institutions are crucial and that democracy leads to high income. There is evidence in this direction: see, for example, Madsen, Raschky and Skali (2015), Cervelatti et al. (2014) and Acemoglu et al. (2019). We do not have a great deal to say about this: it is unlikely that future democracy influences current income—that would contradict our model. Moreover, we have nothing to say about the frontier itself. Our model is not about why some some nations are richer than others; just why poorer ones are more prone to fighting and dominant party rule. Our frontier countries we class as voting countries and it may well be that to have the innovation needed on the frontier a high level of inclusiveness is needed. Certainly playing catch-up is different: Pakistan, a country that is far from inclusive, nevertheless commands the quality and number of engineers needed to develop and produce nuclear weapons.

It is, in fact, consistent with our theory that voting improves economic efficiency. A case-study is Mongolia. Below in Figure 12 is the polyarchy index (the solid curve) normalized so that a country is classed as voting when the index exceeds one, and contemporaneous productive inefficiency normalized so that a country has (according to our estimates) a $50-50$ chance of determining power by voting when the index fall below one.

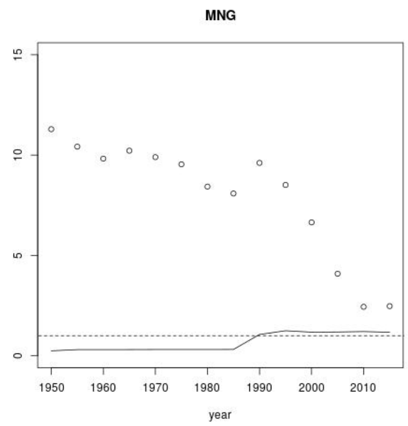

Figure 12: Voting and Productive Inefficiency: Mongolia
Solid curve: polyarchy index/0.6; 0.6 is cutoff for being considered that voting took place. Dots: contemporaneous productive inefficiency $\tilde{\lambda}/2.0$; 2.0 is estimated $\tilde{\lambda}$ for fifty percent chance that voting will take place. Horizontal dotted line: level at one.

The story we tell is this. Mongolia was a communist and a poor country through 1990. At that time the fall of the Soviet Union made the dominant communist party more pessimistic about the cost of fighting and a new constitution was written, both democratic and guaranteeing free market reforms. Power did indeed change hands peacefully with the communist party losing power in 1993 elections. Through the lense of our theory, a shock to expectations lead to transition to a voting regime. As can be seen either this (or the free market reforms) led to rapid catch-up to the frontier, reinforcing (according to our theory) the chances of remaining democratic.

It turns out, however, that Mongolia is rather exceptional. We took the period 1940 to 2015. We selected the countries that were initially far from the frontier in the sense that the maximum productive inefficiency $\tilde{\lambda}$ at or before 1960 exceeded 9, but caught up substantially in the sense that productive inefficiency in 2015 was below 5. There are seven such countries: Brazil (BRA), China (CHN), South Korea (KOR), Romania (ROU), Thailand (THA), Turkey (TUR), and Taiwan (TWN). Below in Figures 14 and 14 are plotted again the normalized voting index and normalized contemporaneous productive inefficiency.

What the data shows is this. China has never been a voting country. Thailand and Turkey just barely for a brief period of time. In all three cases catch-up to the frontier took place under dominant party rule. The remaining countries all saw large declines in productive inefficiency under dominant party rule, and eventual transition to voting about the time that productive inefficiency fell to the level predicting, according to our estimates, about a $50-50$ chance of voting.

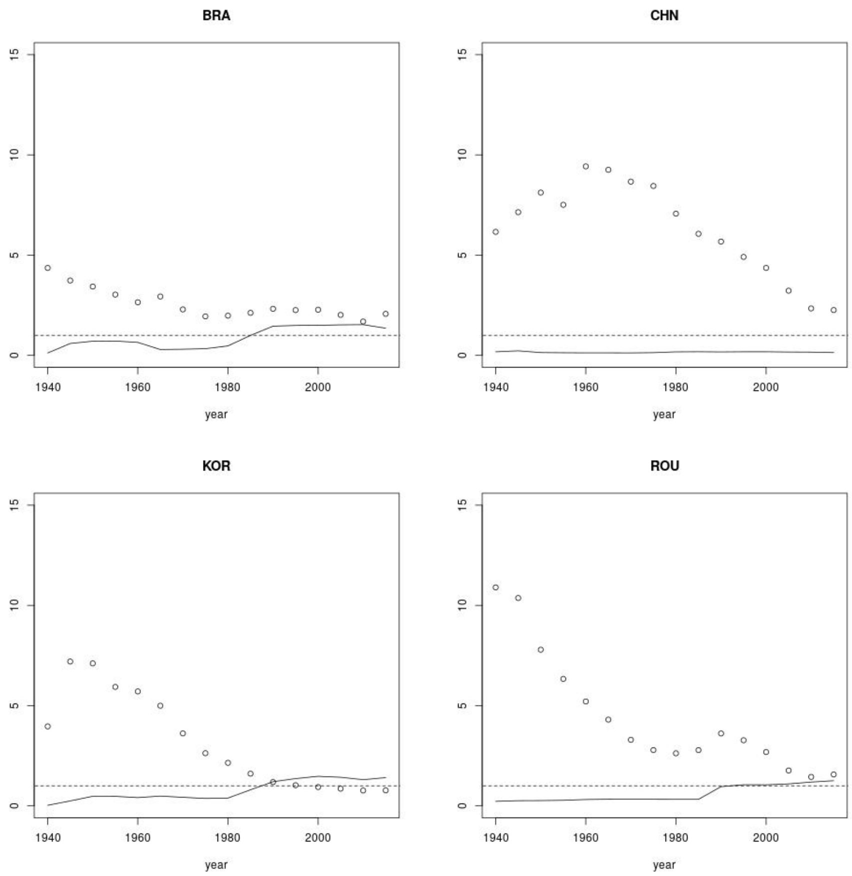

Figure 13: Voting and Productive Inefficiency: Brazil, China, South Korea, Romania
Solid curve: polyarchy index/0.6; 0.6 is cutoff for being considered that voting took place. Dots: contemporaneous productive inefficiency $\tilde{\lambda}/2.0$; 2.0 is estimated $\tilde{\lambda}$ for fifty percent chance that voting will take place. Horizontal dotted line: level at one.

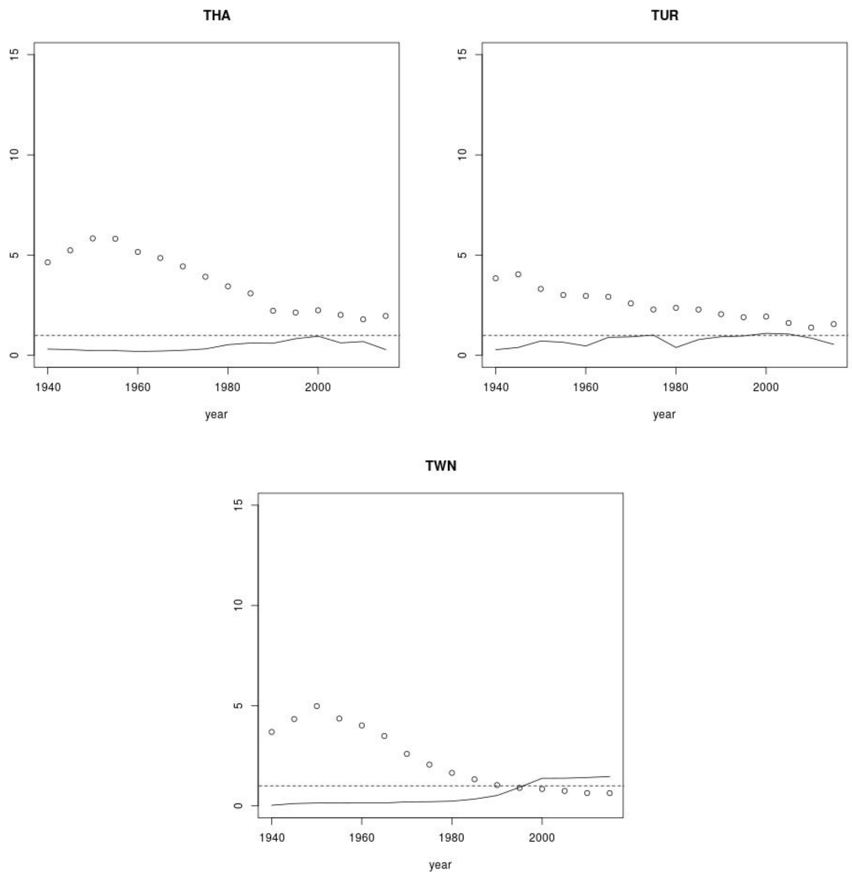

Figure 14: Voting and Productive Inefficiency: Thailand, Turkey, Taiwan
Solid curve: polyarchy index/0.6; 0.6 is cutoff for being considered that voting took place. Dots: contemporaneous productive inefficiency $\tilde{\lambda}/2.0$; 2.0 is estimated $\tilde{\lambda}$ for fifty percent chance that voting will take place. Horizontal dotted line: level at one.

10.3. Cross Country Civil War Regressions

Starting with Collier and Hoeffler (1998) the empirical civil war literature has followed a cross-country regression strategy similar to that used in the study of democracy. The point of departure for most modern work is Fearon and Laitin (2003) who regress the incidence of civil war regressed on its lag, on lagged per capita income and other variables. Parallel to the empirical democracy literature, the main finding is that the likelihood of civil war decreases with income. Chassang and Padro i Miquel (2009) point out two main findings: poorer countries are more likely to suffer from civil war, and civil war is more likely to occur when a country is exposed to a negative income shock. The fact that the likelihood of civil war decreases with income corresponds to our findings. Unlike the literature on democracy there seems to have been no effort to debunk the major finding by using country fixed effects.

11. What Matters for Conflict and Democracy?

In this section we consider factors other than productive inefficiency that might matter for explaining fighting and voting. We first examine whether absolute per capita GDP as measured by the frontier has additional explanatory power. We find that it does not. We then turn two two factors that have been argued as important in the literature: mineral wealth, specifically oil and natural gas, and ethnic division. We find that both are consistent with our model and each has an important effect.

11.1. Absolute Per Capita GDP and Fighting

We have accounted for productive inefficiency, that is, per capita GDP relative to the frontier. Does the frontier itself, that is, absolute per capita GDP, provide additional explanatory power for fighting?

The crucial fact is from section 8.1.1 where we showed that the sample using the data from 1950 and earlier yields essentially the same estimates for fighting as the full sample. In particular, the incidence of fighting is no greater in the early part of the data then in the later part of the data. In the early part of the sample the average height of the frontier was $8,100 while in the later part of the sample it rose by a factor of about four to $37,400. If in fact absolute income mattered, the incidence of fighting should be much lower in the early sample than the entire sample and it is not.

This is analysis, corresponding to the 1 in the introduction is not entirely satisfactory. After 1950 the fraction of poor countries in the sample increases substantially. It is possible that the reason that there is no increase in fighting is that more poor countries increased fighting while the increased per capita GDP decreased fighting and the two effect cancel out leaving no net increase in fighting. Another way to say this, is to observe that in order to estimate the effect of the frontier in a regression that include productive efficiency we must partial out, that is, regress the residuals from the regression of fighting on productive efficiency on the residuals of the regression of the frontier also on productive efficiency, that is, on the component of the frontier that is not explained by productive efficiency.

To check for this, we regressed the frontier on productive efficiency. However, the residuals look very much like the frontier itself: the average rises from $6,600 in the early part of the sample to $33,900 in the later part of the sample. They are plotted in Figure 15 below, where it is clear how sharp is the rise after 1950. Hence the conclusion remain unchanged: if absolute GDP mattered there should be a strong change in the level of conflict between the early sample and the entire sample, and there is not.

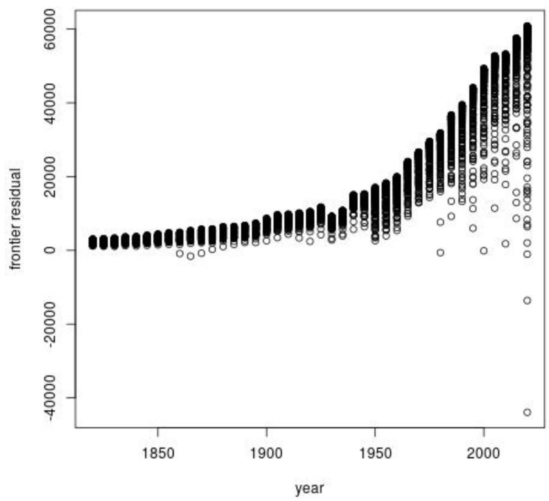

Figure 15: Frontier Residuals

11.2. Intensity of Conflict

We next want to examine the impact of mineral wealth and of ethnic division on fighting and democracy. Our model provides a natural channel through which this can take place: the size of the prize, which so far we have assumed constant across countries and times. We can estimate V on relevant subsamples of countries with high mineral wealth or high ethnic division provided that V is not correlated with λ. Our first step is to show that it is not. The key idea is that we can measure V by the intensity of conflict, so determine whether is is correlated with λ.

As indicated earlier changes in the size of the prize V are reflected linearly in the expected per capita efforts $Ef_k = (1-2Q_0)(\rho_f^{-1}/2)V/C_o$. While the expected effort is not observed, actual per capita effort is. In particular, define $\overline{f} = f_i + f_o$ in per capita terms. Then it should be that $E[\overline{f}|\lambda] = E\overline{f}$ independent of λ. To test if this is the case we can estimate a regression model of the form $E\overline{f}_\tau = \alpha + \beta\lambda_\tau$. The results are shown below in Table 9 and with the scatter-plot in Figure 16.

	Coefficient	SE
α	0.0164	0.0028
β	0.00026	0.00032
Observations	99	

Table 9: Intensity estimation

As can be seen, the slope with respect to λ is slightly positive and small. If the true slope is zero, then there is a 40% probability that such a slope estimate or higher could be generated by sampling error.

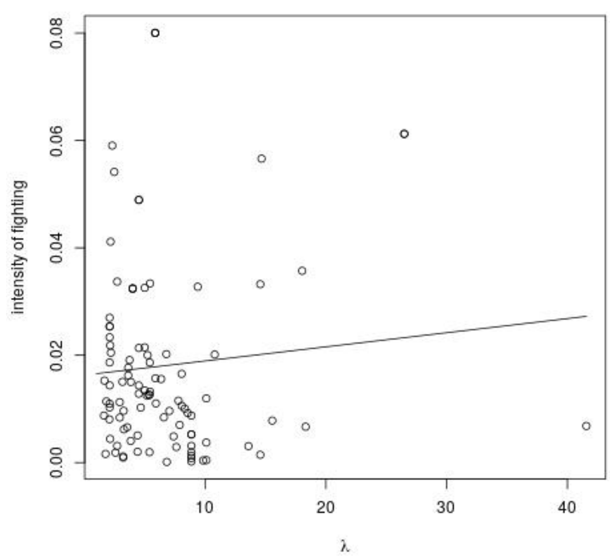

Figure 16: Intensity and Income

The overall conclusion is that there is no convincing evidence that V is correlated with λ. This means that we can legitimately analyze V in subsamples to see if other exogenous variables influence V. Specifically, we estimate equations of the form $Ey_{\ell\tau} = g_\ell(V, \lambda_\tau)$ where $\ell \in \{\text{fight}, \text{novote}\}$ and g_ℓ is the model with parameters estimated from above. We turn now to this analysis.

11.3. Does Oil Matter?

We would expect that oil producing countries would have a larger prize than non-oil producing countries. To assess this we took the subsample of oil producing countries and estimated the size of the prize from the model.[10] Conditional on the parameters estimated for the fighting and voting models, we

[10] We define an oil producing country/period as having oil income at least 5% of GDP. There are 46 countries that met this criterion in at least one period: Angola, Albania, UAR, Argentina, Azerbaijan, Bahrain, Bolivia, Brunei, Canada, Cameroon, Republic of the Congo, Algeria, Ecuador, Egypt, Gabon, Great Britain, Equatorial Guinea, Indonesia, Iran, Iraq, Kazakhstan, Kuwait, Libya, Lithuania, Mexico, Malaysia, Nigeria, Netherlands, Norway, Oman, Peru, Qatar, Russia, Saudi Arabia, South Sudan, Senegal, Suriname, Syria, Chad, Turkmenistan, Trinidad and Tobago, Tunisia, Uzbekistan, Venezuela, and Yemen.

use non-linear least squares to estimate a common value of V for these countries. The results are in the first two columns of Table 10 below.

The theory does well: both for fighting and voting the estimated values of V are much larger than the baseline V of 7.3%. Moreover, the estimate of the fight data is well within two standard errors of the estimate from the vote data. While the estimates are both greater than one, the bottom of the two standard deviation confidence interval from the fight data is 0.60 and the bottom for the vote data is 0.66.

To assess the quantitative importance of this increased V we computed the level of fighting and no-voting predicted by the baseline model for these countries in the third row of 10 and contrasted it with the actual levels of fighting and no voting in the fourth row. As can be seen, fighting is increased by about 9% and no-voting by about 14%. By contrast increasing productive inefficiency from the most productive to least productive group of countries increases fighting by about 18% and no-voting by about 67%. In other words, while oil and natural gas do have a substantial impact on both voting and fighting, productive inefficiency is considerably more important.

11.4. Do Ethnic Divisions Matter?

We would also expect that countries divided along ethnic lines would have a larger prize than more homogeneous countries: the more factionalized the country, the more important it is to win, and subsequently, the more each side stands to gain or lose. To assess this we used the polarization measure from Esteban, Mayoral and Ray (2012) based on language differences and took a subsample of countries for which the index is high.[11]

Conditional on the parameters estimated for the fighting and voting models we again use non-linear least squares to estimate a common value of V for these countries. The results are in the third and fourth column of Table 10 below.

	Oil		Ethnic Division		$\rho_f = 1.4$
	Fighting	No-Voting	Fighting	No-Voting	No-Voting
V	2.46	1.08	4.60	0.12	4.94
SE	0.93	0.23	0.85	0.10	0.21
predicted	0.12	0.78	0.14	0.77	0.29
actual	0.21	0.92	0.27	0.78	
Observations	271	109	393	386	

Table 10: Size of the Prize for Oil and for Ethnic Division

[11] We define a country/period to have high ethnic division if the PEthnoDelta005 index is at least 0.05. There are 44 countries meeting this criterion in at least one period: Afghanistan, UAR, Argentina, Azerbaijan, Bulgaria, Bahrain, Bolivia, Republic of the Congo, Cyprus, Dominican Republic, Ecuador, Estonia, Finland, Fiji, Georgia, Guatemala, Hungary, Iran, Iraq, Israel, Kirghistan, Sri Lanka, Mexico, Myanmar, Mauritania, Mauritius, Malaysia, Niger, Nepal, Oman, Panama, Peru, Paraguay, Romania, Russia, South Sudan, Singapore, Suriname, Slovakia, Syria, Trinidad and Tobago, Turkey, and Vietnam.

Qualitatively the theory does well: both estimates of V are greater than the baseline 0.073. The fighting estimate is much much larger than one, but there is no reason that the consequences of ethnic takeover should be limited by GDP. However, the voting estimate is much smaller than that for the fighting estimate, and indeed the confidence intervals do not overlap.

The discrepancy between fighting and voting can arise in the model if ethnic division impacts an additional parameter besides the size of the prize. It is natural to think that in highly divided countries incumbent advantage is smaller because both sides have private armies on a similar footing. Smaller ρ_f does not change fighting, but it does reduce the incentive of the incumbent to fight and so increases voting. To explore this, the final two columns of Table 10 recompute the no-voting value of V when $\rho_f = 1.4$ rather than the baseline 3.96. As expected this estimate of V is much higher and given the sampling error, essentially the same as that estimated for fighting.

As is the case with oil and natural gas, the impact of ethnic division on fighting is substantial—fighting is increased by 13%, somewhat higher than oil and natural gas, but still substantially less than the 18% due to productive inefficiency. By contrast, there is essentially no effect of ethnic division on voting. In our interpretation this is because while the prize V is higher, decreasing the probability of voting, the incumbent advantage is smaller, so the incumbent is less willing to fight, and hence voting is more likely. These two effects cancel with one another leaving little net change in voting.

12. Conclusion

We built a theoretical model of the substitutability between voting and fighting incorporating the idea that distance from frontier GDP is crucial in explaining both. We showed that this fits well data on both voting and fighting. In particular, though causality runs from income to democracy adoption in our model, this causality link may be missed by empirical methods that are prominent in the current debate on income and democracy. A key takeaway is that reducing global inequality should lead to improved chances that there will be meaningful voting and peaceful transitions of power.

References

Acemoglu, D., S. Johnson, J. A. Robinson, J. A. and P. Yared (2008): "Income and Democracy," *American Economic Review* 98: 808–842.

Acemoglu, D., S. Naidu, P. Restrepo and J. A. Robinson (2019): "Democracy Does Cause Growth," *Journal of Political Economy* 127: 47–100.

Acemoglu, Daron and James A. Robinson (2001): "A Theory of Political Transitions," *American Economic Review* 91: 938–963.

Alcalde, J. and M. Dahm (2007): "Tullock and Hirshleifer: A Meeting of the Minds," *Review of Economic Design* 11: 101–124.

Ashworth, Scott and Ethan Bueno De Mesquita (2009): "Elections with Platform and Valence Competition," *Games and Economic Behavior* 67: 191–216.

Barro, Robert J. (1999): "The Determinants of Democracy," *Journal of Political Economy* 107: S158–S183.

Besley, Timothy J. and Torsten Persson (2008): "The Incidence of Civil War: Theory and Evidence," NBER.

Beviá, C. and L. C. Corchón (2015): "Relative Difference Contest Success Function," *Theory and Decision* 78: 377–398.

Bolt, Jutta and Jan Luiten van Zanden (2020): "Maddison Style Estimates of the Evolution of the World Economy. A new 2020 update," Maddison-Project Working Paper WP-15.

Blattman, C. and E. Miguel (2010): "Civil War," *Journal of Economic Literature* 48: 3–57.

Cervellati, Matteo, Florian Jung, Uwe Sunde, and Thomas Vischer (2014): "Income and Democracy: Comment," *American Economic Review* 104: 707–719.

Che, Y., Y. Lu, Z. Tao and P. Wang (2013): "The Impact of Income on Democracy Revisited," *Journal of Comparative Economics* 41: 159–169.

Chassang, Sylvain, and Gerard Padró i Miquel (2009): "Economic Shocks and Civil War," *Quarterly Journal of Political Science* 4: 211–228.

Collier, Paul, and Anke Hoeffler (1998) "On Economic Causes of Civil War," *Oxford Economic Papers* 50: 563–73.

Corchón, L. C. and A. Yildizparlak (2013): "Give Peace a Chance: The Effect of Ownership and Asymmetric Information on Peace," *Journal of Economic Behavior and Organization* 92, 116–126.

Dal Bo E. and P. Dal Bo (2011): "Workers, Warriors, and Criminals: Social Conflict in General Equilibrium," *Journal of the European Economic Association* 9: 646–677.

Dal Bo, E. and R. Powell (2009): "A Model of Spoils Politics," *American Journal of Political Science* 53: 207–222.

Dixon, Jeffrey S. and Meredith Reid Sarkees (2016): *A Guide to Intra-state Wars: An Examination of Civil, Regional, and Intercommunal Wars, 1816-2014*, Washington, DC: CQ Press 2016.

Dell, M. B. Jones and B. Olsen (2012): "Temperature Shocks and Economic Growth: Evidence from the Last Half Century," *American Economic Journals: Macro* 4: 66–95.

Dupuy, T. N. (1986): "Mythos or Verity? The Quantified Judgment Model and German Combat Effectiveness," *Military Affairs: The Journal of Military History, Including Theory and Technology*: 204–210.

Ewerhart, C. (2017): "Contests with Small Noise and the Robustness of the All-Pay Auction," *Games and Economic Behavior* 105: 195–211.

Esteban, J., L. Mayoral and D. Ray (2012): "Ethnicity and Conflict: An Empirical Study," *American Economic Review* 102: 1310–1342.

Fearon, James D. (1995): "Rationalist Explanations for War," *International Organization* 49: 379–414.

Fearon, J. D. (2011): "Self-Enforcing Democracy," *Quarterly Journal of Economics* 126: 1661–1708.

Fearon, James D., and David D. Laitin (2003): "Ethnicity, Insurgency, and Civil War," *American Political Science Review* 97: 75–90.

Feder, P. I. (1975): "The Log Likelihood Ratio in Segmented Regression," *The Annals of Statistics* 3: 84–97.

Hillman, A. L. and J. Riley (1989): "Politically Contestable Rents and Transfers," *Economics and Politics* 1: 17–39.

Hirshleifer, J. (1989): "Conflict and Rent-Seeking Success Functions: Ratio vs. Difference Models of Relative Success," *Public Choice* 63: 101–112.

Hirshleifer, J., M. Boldrin and D. K. Levine (2009): "The Slippery Slope of Concession," *Economic Inquiry* 47: 197–205.

Kortum, S. S. (1997): "Research, Patenting, and Technological Change," *Econometrica* 65: 1389–1419.

Laurent-Lucchetti, Jérémy, Dominic Rohner, and Mathias Thoenig. "Ethnic Conflict and the Informational Dividend of Democracy," *Journal of the European Economic Association* 22: 73–116.

Levine, D. K. and A. Mattozzi (2020): "Voter Turnout with Peer Punishment," *American Economic Review* 110: 3298–3314.

Levine, D. K. and A. Mattozzi (2022): "Success in Contests," *Economic Theory* 73: 595–624.

Levine, D. K. (1985): "Sensitivity of MLE to Measurement Error," *Journal of Econometrics* 28: 223–230

Levine, D. K., A. Mattozzi, and S. Modica (2022): *Social Mechanisms and Political Economy: When Lobbyists Succeed, Pollsters Fail and Populists Win*, mimeo EUI.

Levine, D. K. and S. Modica (2018): "Intervention and Peace," *Economic Policy* 33: 361–402.

Lipset, Seymour M. (1959): "Some Social Requisites of Democracy: Economic Development and Political Legitimacy," *American Political Science Review* 53: 69–105.

Madsen, J. B., P. A. Raschky and A. Skali (2015): "Does Democracy Drive Income in the World, 1500–2000?" *European Economic Review* 78: 175–195.

Mawejje, J. and P. McSharry (2021): "The Economic Cost of Conflict: Evidence from South Sudan," *Review of Development Economics* 25: 1969–1990.

Miguel, E., S. Satyanath and E. Sergenti (2004): "Economic shocks and civil conflict: An instrumental variables approach," *Journal of Political Economy* 112: 725-753.

Pemstein, Daniel, Kyle L. Marquardt, Eitan Tzelgov, Yi-ting Wang, Juraj Medzihorsky, Joshua Krusell, Farhad Miri, and Johannes von Römer (2024): "The V-Dem Measurement Model: Latent Variable Analysis for Cross-National and Cross-Temporal Expert-Coded Data," V-Dem Workimg Paper.

Przeworski, A., G. Rivero and T. Xi (2015): "Elections as a conflict processing mechanism," *European Journal of Political Economy* 39: 235–248.

Ross, Michael and Paasha Mahdavi (2015), "Oil and Gas Data, 1932–2014," Harvard Dataverse.

Tullock, G. (1967): "The Welfare Costs of Tariffs, Monopolies, and Theft," *Western Economic Journal* 5: 224–232.

Tullock, G. (1980): "Efficient Rent Seeking," In J. M. Buchanan, R. D. Tollison and G. Tullock (Eds.), *Towards a Theory of the Rent-seeking Society*: 97–112, Texas A&M University Press.

Online Appendix 1: Measurement Error

Let $\varphi \geq 1$ be the ratio of combatants and $\tilde{\varphi} = \eta\varphi$ the actual force ratio where the multiplicative shock η is independent of the ratio of combatants φ and η has support bounded above and away from zero. Suppose the support of φ is unbounded above and denote the joint density by $f(\tilde{\varphi}, \varphi)$.

Lemma 1. *The positive correlation conditions*

$$\lim_{\overline{\varphi}\to\infty} \Pr\left(\tilde{\varphi} \leq \sqrt{\overline{\varphi}}|\varphi \geq \overline{\varphi}\right) = 0$$

and

$$\lim_{\overline{\varphi}\to\infty} \frac{\Pr\left(\tilde{\varphi} \leq \sqrt{\overline{\varphi}}, \varphi > \overline{\varphi}\right)}{\Pr\left(\tilde{\varphi} > \sqrt{\overline{\varphi}}, \varphi > \overline{\varphi}\right)} = 0$$

hold.

Proof. For $\eta\varphi \leq \sqrt{\overline{\varphi}}$ to be true it must be true that $\overline{\varphi} \leq \sqrt{\overline{\varphi}}/\underline{\eta}$ which fails as $\overline{\varphi} \to \infty$. Hence the first positive correlation condition is satisfied. Next

$$\lim_{\overline{\varphi}\to\infty} \frac{\Pr\left(\eta\varphi \leq \sqrt{\overline{\varphi}}, \varphi > \overline{\varphi}\right)}{\Pr\left(\eta\varphi > \sqrt{\overline{\varphi}}, \varphi > \overline{\varphi}\right)}$$

$$\leq \lim_{\overline{\varphi}\to\infty} \frac{\Pr\left(\underline{\eta}\varphi \leq \sqrt{\overline{\varphi}}, \varphi > \overline{\varphi}\right)}{\Pr\left(\overline{\eta}\varphi > \sqrt{\overline{\varphi}}, \varphi > \overline{\varphi}\right)}.$$

For $\overline{\varphi}$ sufficiently large the numerator is 0 and the denominator 1. □

Denote by $0 \leq Q(\tilde{\varphi}) \leq 1/2$ is the probability of success and suppose that $\lim_{\tilde{\varphi}\to\infty} Q(\tilde{\varphi}) = Q_0$. Let $Q_T(\overline{\varphi})$ denote the frequency of success for $\tilde{\varphi} > \overline{\varphi}$ in a sample of size T.

Proposition 1. *There exists a sequence $\overline{\varphi}_T$ such that $Q_T(\overline{\varphi}_T)$ converges in probability to Q_0....*

Proof. For fixed $\overline{\varphi}$ the fact that φ has unbounded support implies that

$$\text{plim}_{T\to\infty} Q_T(\overline{\varphi}) = \frac{\int_{\overline{\varphi}}^{\infty} \int_1^{\infty} Q(\tilde{\varphi}) f(\tilde{\varphi}, \varphi) d\tilde{\varphi} d\varphi}{\int_{\overline{\varphi}}^{\infty} \int_1^{\infty} f(\tilde{\varphi}, \varphi) d\tilde{\varphi} d\varphi}.$$

For any $\overline{\varphi}$ we may write

$$\text{plim}_{T\to\infty} Q_T(\overline{\varphi}) = \frac{\int_{\overline{\varphi}}^{\infty} \int_{\sqrt{\overline{\varphi}}}^{\infty} Q(\tilde{\varphi}) f(\tilde{\varphi}, \varphi) d\tilde{\varphi} d\varphi + \int_{\overline{\varphi}}^{\infty} \int_1^{\sqrt{\overline{\varphi}}} Q(\tilde{\varphi}) f(\tilde{\varphi}, \varphi) d\tilde{\varphi} d\varphi}{\int_{\overline{\varphi}}^{\infty} \int_1^{\infty} f(\tilde{\varphi}, \varphi) d\tilde{\varphi} d\varphi} \equiv \hat{Q}(\overline{\varphi}).$$

We compute the difference between Q_0 and $\hat{Q}(\overline{\varphi})$ in three steps.

First step:

$$\frac{\int_{\overline{\varphi}}^{\infty}\int_{1}^{\sqrt{\overline{\varphi}}} f(\tilde{\varphi},\varphi)d\tilde{\varphi}d\varphi}{\int_{\overline{\varphi}}^{\infty}\int_{1}^{\infty} f(\tilde{\varphi},\varphi)d\tilde{\varphi}d\varphi} = \Pr\left(\eta\varphi \leq \sqrt{\overline{\varphi}}\,|\varphi \geq \overline{\varphi}\right)$$

$$\leq \Pr\left(\underline{\eta}\varphi \leq \sqrt{\overline{\varphi}}\,|\varphi \geq \overline{\varphi}\right)$$

which is less than $\epsilon/3$ for some large $\overline{\varphi}_{\epsilon}^{1}$ and $\overline{\varphi} > \overline{\varphi}_{\epsilon}^{1}$ by the first correlation condition in Lemma 1.

Second step: For some large $\overline{\varphi}_{\epsilon}^{2}$ and $\overline{\varphi} > \overline{\varphi}_{\epsilon}^{2}$ since $\tilde{\varphi} \geq \underline{\eta}\varphi$ we also have have $|Q(\overline{\varphi}) - Q_0| < \epsilon$.

Third step:

$$\frac{\int_{\overline{\varphi}}^{\infty}\int_{\sqrt{\overline{\varphi}}}^{\infty} f(\tilde{\varphi},\varphi)d\tilde{\varphi}d\varphi}{\int_{\overline{\varphi}}^{\infty}\int_{1}^{\infty} f(\tilde{\varphi},\varphi)d\tilde{\varphi}d\varphi}$$

$$= \frac{\int_{\overline{\varphi}}^{\infty}\int_{\sqrt{\overline{\varphi}}}^{\infty} f(\tilde{\varphi},\varphi)d\tilde{\varphi}d\varphi}{\int_{\overline{\varphi}}^{\infty}\int_{\sqrt{\overline{\varphi}}}^{\infty} f(\tilde{\varphi},\varphi)d\tilde{\varphi}d\varphi + \int_{\overline{\varphi}}^{\infty}\int_{1}^{\sqrt{\varphi_T}} f(\tilde{\varphi},\varphi)d\tilde{\varphi}d\varphi}$$

$$= \left(1 + \frac{\int_{\overline{\varphi}}^{\infty}\int_{1}^{\sqrt{\overline{\varphi}}} f(\tilde{\varphi},\varphi)d\tilde{\varphi}d\varphi}{\int_{\overline{\varphi}}^{\infty}\int_{\sqrt{\overline{\varphi}}}^{\infty} f(\tilde{\varphi},\varphi)d\tilde{\varphi}d\varphi}\right)^{-1}.$$

We have

$$\frac{\int_{\overline{\varphi}}^{\infty}\int_{1}^{\sqrt{\overline{\varphi}}} f(\tilde{\varphi},\varphi)d\tilde{\varphi}d\varphi}{\int_{\overline{\varphi}}^{\infty}\int_{\sqrt{\overline{\varphi}}}^{\infty} f(\tilde{\varphi},\varphi)d\tilde{\varphi}d\varphi} = \frac{\Pr\left(\eta\varphi \leq \sqrt{\overline{\varphi}}, \varphi > \overline{\varphi}\right)}{\Pr\left(\eta\varphi > \sqrt{\overline{\varphi}}, \varphi > \overline{\varphi}\right)}$$

and for sufficiently large $\overline{\varphi}_{\epsilon}^{3}$ and $\overline{\varphi} > \overline{\varphi}_{\epsilon}^{3}$ this goes to zero by the second correlation condition in Lemma 1. Hence

$$\frac{\int_{\overline{\varphi}}^{\infty}\int_{\sqrt{\overline{\varphi}}}^{\infty} f(\tilde{\varphi},\varphi)d\tilde{\varphi}d\varphi}{\int_{\overline{\varphi}}^{\infty}\int_{1}^{\infty} f(\tilde{\varphi},\varphi)d\tilde{\varphi}d\varphi} \to 1.$$

Taking $\overline{\varphi}_{\epsilon} = \overline{\varphi}_{\epsilon}^{1} + \overline{\varphi}_{\epsilon}^{2} + \overline{\varphi}_{\epsilon}^{3}$ we see that for $\overline{\varphi} \geq \overline{\varphi}_{\epsilon}$we have $|\hat{Q}(\overline{\varphi}) - Q_0| \leq \epsilon$. For each ϵ choose T such that $\Pr\left(|Q_T(\overline{\varphi}_{\epsilon}) - \hat{Q}(\overline{\varphi})| > \epsilon\right) < \epsilon$ and define $\overline{\varphi}_T = \overline{\varphi}_{\epsilon}$.... Letting $\epsilon \to 0$ now gives the desired result. □